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Managing Risk: The Portfolio Benefits of Global Diversification

A Thesis in Economics

By

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Abstract

This thesis looks at some of the different ways risk is measured in the financial markets and traces the evolution of the autoregressive conditional heteroskedasticity (ARCH) class of models. In addition, this thesis proposes a new way to manage risk using international diversification by comparing the risk characteristics of United States headquartered companies based on whether they earn the majority of their revenue domestically or internationally and indices that track both the United States market and global markets. Furthermore, these risk characteristics are studied over three different periods—the pre-financial crisis period, financial crisis period, and post-financial crisis period, which gives the added benefit of studying how risk changes during times of extreme market stress. Ultimately, the companies that generated significant revenue from overseas performed better on the majority of the measures analyzed, especially during the financial crisis.

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1. Introduction

This thesis seeks to study risk and the different ways risk is measured within finance, which is prevalent given the volatile start to 2016 for many financial markets around the world. January of 2016, in particular, was a volatile month for markets around the world. The selling began on the first trading day of the year and continued for the majority of the month leading to the worst start to a year in history for U.S. stocks. Concerns over slowing growth in China—the world's second largest economy—were rekindled from August 2015 by a persistent drop in the renminbi and substantial drops in the Chinese equity markets that triggered circuit breakers, resulting in early endings to two out of first four trading days. The selling continued throughout the month as a plunge in oil prices raised additional concerns about slowing global growth, deflation, and bankruptcies among oil companies. As a result, the correlation between the S&P 500 and Brent crude oil reached its highest level in 26 years,[1] meaning that oil and stocks were moving in the nearly the same proportion and the same direction. This type of correlation is rare, except during recessions, and signals that investors were less enthusiastic about holding risky assets. Finally, uncertainty over the future path of interest rate increases by the Federal Reserve helped compound the selling. Although many investors do not believe the Fed should be increasing rates in this type of global environment, several speeches by Fed members reiterated their plans to increase rates four times this year, which only

[1] Stubbington, Tommy and Georgi Kantchev. "Oil, Stocks in Tightest Lockstep in 26 Years." *Wall Street Journal*, January 25, 2016.

contributed to the uncertainty. All of these factors contributed to a volatile January in which investor risk appetites decreased.

In light of these factors, studying risk and its characteristics has become increasingly important for financial market participants. In addition to providing an overview of risk in general, this thesis explores a new approach to managing risk by studying how company risk changes based on the underlying company. Specifically, using the renowned generalized autoregressive conditional heteroskedasticity [GARCH(1,1)] model developed by Bollerslev in 1986, the risk characteristics of companies that obtain the majority of their revenue within the United States are analyzed to see if they differ from companies that generate a significant portion of their revenue from outside the United States. The goal of doing so is to determine if the U.S. economy provides advantages great enough to warrant higher investment there or if investors could potentially use U.S. headquartered companies that are themselves internationally diversified to better manage their own investment risk. Additionally, the S&P 500 is compared to the MSCI All Country World Index—a proxy for the global stock market—to see if international diversification is better attained by investing in an index. Risk characteristics for each basket of companies and both indices are also compared across different time periods. The time period leading up to the financial crisis (2005 to early-2007), the financial crisis (early 2007 to early 2009), and the post financial crisis period (early-2009 to 2014) are considered, in particular. Analyzing the risk characteristics of different companies across different time periods provides the added benefit of studying how volatility changes during times of market crisis compared to more normal times. All of these factors are fundamentally important to traders, investors, and policy makers alike. Whether it is an investor is looking to construct an optimal portfolio or an options trader is looking to take advantage of increased volatility speculate on an asset, a superior volatility forecast can help all market participants and regulators prepare for the uncertainty of the future.

Using Bollerslev's GARCH(1,1) model in conjunction with other risk metrics developed in section two, it becomes apparent that the portfolio comprised of companies that generate significant revenue from outside the United States outperformed all the other portfolios, suggesting that there are benefits to international diversification. This also suggests that there are other avenues available to investors who wish to diversify internationally. Instead of having to own foreign equities, they could instead look to own companies that are headquartered in the United States, but are themselves diversified internationally. While looking at the two indices, the MSCI ACWI displayed lower volatility than the S&P 500 during the financial crisis despite outperforming horribly throughout the rest of the analysis, providing further evidence for international diversification.

The rest of this paper is structured as follows. Section 2 looks at how we define risk. It establishes the foundation for quantifying risk in finance and traces the evolution of the autoregressive conditional heteroskedasticity (ARCH) class of models. Section 3 looks at why there seems to be a lack of international diversification among investors by introducing the home bias puzzle. It also poses the question of how we might apply risk models to help us learn more about international diversification by introducing portfolios comprised of U.S. headquartered companies that earn the majority of their revenue domestically, U.S. headquartered companies that earn a large portion of their revenue from overseas, and various index based portfolios. Finally, section 3 introduces the idea of managing risk by using international diversification, without actually having to actually buy international equities. Section 4 looks at some performance statistics of the different portfolios constructed in section 3, implements a GARCH(1,1) model to answer the questions posed in section 3, and discusses some of the interesting revelations seen through the model. Section 5 concludes and poses questions for future research.

2. How Risk Is Defined in Finance

2.1 – A Bird's Eye View of Risk

Our ability to conceptualize and analyze risk is arguably the building block upon which modern society is built. One would think that the spectacular advancements of medicine, technology, and economic and political progress is what separates us from our predecessors that lived thousands of years ago. Instead, it is our ability to evaluate future outcomes, differentiate between other possible outcomes, and make good, informed decisions based on the information we have.[2] Analyzing risk helps governments set public health standards, decide where to distribute funds, and make decisions about whether or not to engage in war. It helps companies raise money to fund their growth and helps them decide whether or not to take on specific projects. It helps individuals allocate wealth and determine whether or not they should buy a new house or a car. Without a

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[2] Peter L. Bernstein, *Against the Gods: The Remarkable Story of Risk* (New York: John Wiley & Sons, Inc., 1996), 1-2.

proper way to analyze risk, insurance companies might not exist. Consequently, families might not buy cars or homes due to the threat of destruction or the death of a provider might leave the rest of the family unable to afford food or other necessities. Without a good way to analyze risk, only well off families could afford to buy homes or pay for health care. Food producers would not plant as much if they had no way of knowing what price they could sell their crops for. Our roads and public transportation might not exist if engineers had no way of understanding the risks associated with building highways, bridges, and train tracks. Without risk analysis, perhaps we would not have won World War II. We wouldn't have put a man on the moon. Diseases that existed centuries ago would still plague us. Risk is everywhere in our modern day society. As individuals, governments, and companies we constantly have to make decisions about the future and, without effective methods for doing so, society would not have made the progress it has.

Similarly, financial market participants and policy makers need to assess future outcomes before making important decisions. As a result, modeling risk has long been a key undertaking by both academics and market participants. Typically, risk is thought of as a negative. This view is not completely accurate, however. Former president John F. Kennedy, who was still a Senator at the time, popularized the notion that the Chinese word for crisis is made up of two characters—one representing danger and the other opportunity.[3] Although Senator Kennedy was referring to the dangers posed by the Soviet Union and the opportunities created by the space age, we can still approach risk in the

[3] John F. Kennedy, "Remarks of Senator John F. Kennedy (Dem. – Mass.) at the 1959 Convocation of the United Negro College Fund" (speech, Indianapolis, Indiana, April 12, 1959), John F Kennedy Presidential Library, http://www.jfklibrary.org/Asset-Viewer/To6xnVCeNUSecmWECy7Fpw.aspx.

financial markets in a similar fashion—as a combination of both opportunity and danger. In other words, we can think of risk in the sense that asset prices move differently than what people are expecting. The fact that risk is presented as this type of uncertainty provides the opportunity to create different methods to model it, most of which utilize the concept of volatility as an important input, that can help make investment decisions, create portfolios, price derivatives, comply with regulatory demands, and even make monetary policy decisions. For instance, most investors have pre-determined levels of risk that they are either willing or allowed to take on. Therefore a proficient volatility forecast over the lifespan of the investment can serve as a valuable tool for evaluating investment risk and constructing safe, well-balanced portfolios. Markowitz (1952) proposed that expected portfolio returns and the variance of those returns were factors for constructing optimal portfolios. The variance of the expected portfolio return is based on the weighted variances and covariances of the securities in the portfolio. For this reason, accurate volatility forecasts can help portfolio managers construct favorable portfolios.

Volatility is a crucial component in the pricing of most derivative securities, which have become immensely more popular in the past few decades. As of December 2013, the total notional outstanding in the OTC derivatives market was over $710 trillion, representing a gross market value of almost $19 trillion. In December 1999, these numbers were $88 trillion and $2.8 trillion respectively.[4] The price of an option, for example, is dependent in part on the volatility traders are expecting in the underlying security. In fact, the price of an option can fluctuate purely based on the implied volatility

[4] Bank For International Settlements, "Triennial Central Bank Survey." Basel, 2007.

of the option, even as the price of the underlying remains stable. A simple example of this comes from the equity markets before a company reports earnings. It is almost always the case that the implied volatility on call and put options will rise going into an earnings report[5]—which increases the price of the option—even if the price of the underlying stock does not move significantly. The simple expectation that prices will move following the report causes the option to become more expensive.

Modeling volatility is also an essential component for helping institutions and regulatory officials to ensure stability in the financial sector. After the collapse of Breton Woods in 1973 and the ensuing market turbulence that caused several large banks around the world to face substantial foreign exchange related losses, 11 nations came together to form the Basel Committee.[6] The Basel Committee was created both to strengthen regulation, supervision, and the practices of banks across the globe and to enhance financial stability worldwide. Ever since the inception of the Basel Accords in 1996, volatility forecasting has become increasingly important in a risk management sense for financial institutions throughout the world, as they seek to comply with regulators. For example, the Revised Capital Charge in Basel III requires banks to meet capital requirements expressed as a sum of

> "higher of (1) previous day's value-at-risk number (VaR (-1)) and (2) average of daily value-at-risk measures on each of preceding sixty business days (VaRavg), multiplied by multiplication factor (mc), plus higher of (1) latest available stressed-value-at-risk number above (sVaR (-1)) and (2) an average of stressed value-at-risk numbers over the

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[5] Isakov, Dušan and Christophe Pérignon, "Evolution of Market Uncertainty Around Earnings Announcements," *Journal of Banking and Finance* 25.9 (2001): 1774.
[6] Bank For International Settlements, "A Brief History of the Basel Committee." Basel, 2015.

> preceding sixty business days (sVaRavg), multiplied by multiplication factor (ms)."[7]

To achieve such value at risk (VaR) calculations, a good volatility forecast is necessary, in addition to other factors.

Given that volatility in the financial markets can have a direct effect on the performance of the economy, it is often useful for policy makers and central bankers to look at volatility estimates to see if any weaknesses in the financial markets could spill over into the broader economy. In April of 2007, New Century Financial, one of the top subprime mortgage lenders, filed for chapter 11-bankruptcy protection.[8] A little over a month later, Chairman Ben Bernanke of the Federal Reserve stated that the Fed believed "the effect of the troubles in the subprime sector on the broader housing market will be limited and we do not expect significant spillovers from the subprime market to the rest of the economy or to the financial system."[9] Perhaps a better volatility forecast might have provided evidence to the contrary. In the summer months that followed, Bear Stearns liquidated two hedge funds that invested in different mortgage backed securities. It was clear by then that trouble was beginning to spread to other banks around Wall Street. Finally, in August of 2007, the Federal Reserve began to intervene and cut the rate at which it lends to banks.[10] By this point it was too late, however. Liquidity was disappearing fast and the year that followed was one of the most turbulent in the history

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[7] Latham & Watkins, "Regulatory Capital Reform Under Basel III." New York, 2011.
[8] Federal Reserve Bank of St. Louis, "The Financial Crisis: Full Timeline." St. Louis, 2011.
[9] Associated Press, "Bernanke: Subprime Mortgage Woes Won't Seriously Hurt Economy." *CNBC*, May 17, 2007.
[10] Federal Reserve Bank of St. Louis, "The Financial Crisis: Full Timeline." St. Louis, 2011.

of the financial markets. As a result, the United States—and several other countries around the world—fell into the worst recession since the Great Depression.

2.2 – Some Basic Terminology

At this point, the question of how to quantify the massive uncertainties of the future and make informed forecasts about the markets emerges. Before we can fully understand and appreciate risk—and move onto studying more sophisticated models for forecasting volatility—we must start by comprehending the building blocks for measuring risk, the simplest of which is the unconditional standard deviation. Standard deviation (σ)—or variance—(σ^2) is frequently described as volatility and is computed from a set of data values as

$$\sigma^2 = \frac{1}{N}\sum_{t=1}^{N}(R_t - \bar{R})^2 \tag{1}$$

where $\bar{R}$ is the mean return and R_t is the return in the current period.[11] Standard deviation measures the amount of variation the data values have around the mean. A number closer to zero would show that the data values are close to the mean, on average, while a higher number shows that the data points are farther away from the mean. The drawback to using standard deviation to measure volatility is that it places equal weight on each observation and fails to take into account the fact that price returns are not constant across time. Assuming that returns follow a normal distribution, standard deviation would

[11] Paul Newbold, *Statistics for Business and Economics* (Boston: Pearson Education Inc., 2013), .

be the correct measure of dispersion or variation. However, this is not always the case and there are better measures and models of volatility when normality is not present.

One popular metric in finance that makes use of standard deviation is the Sharpe ratio, which indicates the excess return of a portfolio or asset over that of a risk free rate or benchmark return per unit of risk associated with that excess return.[12] If we let R_{At} be the return of a portfolio or security in the period t and R_{Ft} the return of a "risk free" security, then the excess return for that period, D_t, is

$$D_t = R_{At} - R_{Ft} \tag{2}$$

We can calculate $\overline{D}$, which is the average value of D_t from the period t=1 through t=T. Using equation (1) on D_t and $\overline{D}$ gives us the standard deviation of the excess returns (σ_D). We can then use this to calculate the historic Sharpe ratio for a portfolio or security (S_h) as follows:

$$S_h = \frac{\overline{D}}{\sigma_D} \tag{3}$$

The ex post Sharpe ratio shown in equation (3) measures a portfolio or asset's historical return over a riskless asset per unit of risk employed.[13] This is especially important to portfolio managers who strive to beat the Sharpe ratio of their respective benchmarks. With favorable strategies aimed at lowering a portfolio's volatility, such as the one presented in section 4, portfolio managers can achieve this goal.

Another, more advanced, approach for measuring volatility makes use of the derivatives market. Implied standard deviation (ISD) estimates the volatility of the price

[12] Sharpe, William, "The Sharpe Ratio," *The Journal of Portfolio Management* 21.1 (1994): 51.
[13] Sharpe, William, "The Sharpe Ratio," *The Journal of Portfolio Management* 21.1 (1994): 52-54

of an option, given all the other variables in the Black Scholes (BS) option-pricing model. The BS formula for a call option is

$$C_s = SN(d_1) - Ke^{-rt}N(d_2) \tag{4}$$

where $d_1 = \frac{\ln\left(S/_{Ke^{-rt}}\right)+1/2\sigma^2 t}{\sigma\sqrt{t}}$ and $d_2 = d_1 - \sigma\sqrt{t}$. In equation (2), C_s is the price of the call option, S is the stock price, K is the strike price, r is the continuously compounded annualized risk free rate, t is the time to expiration in years, σ is the annualized standard deviation of the instantaneous rate of return on the underlying, and N is the cumulative standard normal density function.[14] Using the BS model as a starting point, Brenner and Subrahmanyam (1988) determine that the value of an at the money straddle[15] is equal to

$$STR_S = C_S + P_S = 2[0.398S\sigma\sqrt{t}] \tag{5}$$

and therefore

$$\sigma = \frac{STR_S}{2(0.398S\sqrt{t})} \tag{6}$$

by rearranging equation (5).[16] So, for example, a stock trading a \$99.50 with 3 months until expiry and a \$100 strike call option price of \$4.50 and \$100 strike put option price of \$5.00 would yield an implied standard deviation of $\sigma = \frac{4.5+5}{2\times0.398\times100\times\sqrt{0.25}} = 0.24$ using equation (4).

The major breakthrough provided by the Black Scholes model was the novel approach to managing risk. As economic theory changed in the years prior to their

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[14] Black, Fischer and Myron Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81.3 (1973): 642-644.

[15] A straddle is equal to a call option and put option taken together

[16] Brenner, Menachem and Marti G. Subrahmanyam. "A Simple Formula to Compute the Implied Standard Deviation." *Financial Analysts Journal* 44.5 (1988): 80-83.

seminal work, so did the approaches for modeling and managing the risk of portfolios. Black and Scholes ultimately provided a solution to this evolving theory. They determined that the best way to manage risk was by hedging directional positions through the use of options. This breakthrough provided all investors and traders in the marketplace the ability to hedge their portfolios, which also ends up giving the added benefit of yielding additional information on the future expected volatility of certain securities. With more traders and investors participating in a market, the efficiency of the pricing in that market increases. As a result, the ISD method is an increasingly popular model for forecasting volatility that is more appropriate to use than a simple historical standard deviation given the weight it inherently places on new information. Similar to Black Scholes, this thesis also provides a new approach to managing risk. Instead of managing it through hedging, however, this thesis explores the possibility of diversifying away risk through an original approach.

A popular extension of this ISD model is the volatility index, or VIX, constructed by the Chicago Board Options Exchange (CBOE). The VIX was introduced in 1993 and is constructed to be a representation of the 30-day implied volatility by using at the money S&P options.[17] Originally, the VIX was calculated by taking a weighted average of four just in and just out of the money of the money puts and calls on the S&P 100 index. In 2003, the CBOE updated the VIX to provide the implied volatility on the S&P 500 using a wider range of weighted calls and puts. Since the VIX has a constant 30-day maturity and measures the implied 30-day volatility on the S&P 500, the CBOE began

[17] Chicago Board Options Exchange, "The CBOE Volatility Index – VIX." Chicago, 2014.

using weekly S&P options in 2014 to more accurately maintain the desired 30-day maturity. Equations (7) and (8) show how the VIX is currently constructed.

$$VIX = 100\sigma \tag{7}$$

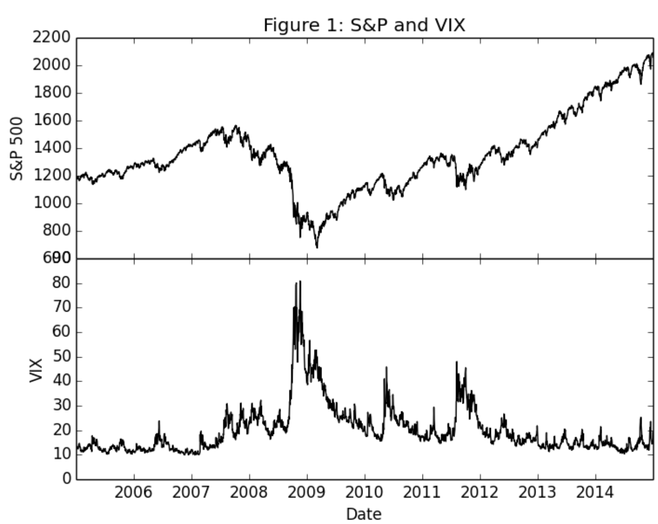
and

$$\sigma = \sqrt{\frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{rt} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2} \tag{8}$$

where F is the forward index level desired from index option prices, K_0 is the first strike below the forward index level (F), K_i is the strike price of the i^{th} out of the money option (a call if $K_i > K_0$, a put if $K_i < K_0$, and both a call and a put if $K_i = K_0$), ΔK_i is the interval between strike prices ($\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$), and $Q(K_i)$ is the midpoint of the bid-ask spread for each option with strike K_i.[18]

The VIX is frequently cited by the mainstream media and is commonly referred to as the "fear" index since it measures investor and trader uncertainty over the next month. In periods of low volatility, the VIX can drop to levels that could signal complacency in the markets. In times of crisis, on the other hand, the VIX will spike above its "normal" level. If

Fig. 1. The S&P 500 vs. the Volatility Index. Data is daily closing prices from January 2005 to December 2015. *Source:* Yahoo! Finance



[18] Chicago Board Options Exchange, "The CBOE Volatility Index – VIX." Chicago, 2014.

uncertainty is exceptionally high, the VIX will remain elevated for a period of time until investors change their outlooks on the future. A clear, recent example of this came during the 2007-2009 financial crisis. Figure 1 shows that, in the latter half of 2007, volatility began to rise. This coincided with the S&P 500 peaking and the collapse of two Bear Stearns hedge funds that were heavily invested in mortgage-backed securities. Although volatility remained fairly constant for the next few months, it remained higher than it had been in the years prior to 2007, signaling heightened risk aversion among stock market participants. When the latter half of 2008 hit, we see a massive spike in the VIX combined with the S&P 500 plummeting from around 1200 to below 700. This was the time period where Lehman Brothers went bankrupt and investors expected that many other large banks would soon follow. The VIX went over 80 for a brief period in 2008, meaning that the expected one-year volatility for the S&P 500 was over 80% at a 68% confidence interval. This is a huge number compared to the typical annual returns of the S&P. Even as the market bottomed in 2009, volatility remained elevated above normal levels. Clearly the events of the previous year were still on the minds of investors and that collective view kept the VIX at levels higher than it had been during the previous bull market.

2.3 – Building Up to GARCH

Before arriving at the full-blown ARCH model and the generalized version of it, it is beneficial to first review the simple linear regression model and autoregressive model. A simple bivariate linear regression shows a relationship between two variables, x and y:

$$y = \beta_0 + \beta_1 x + \varepsilon \tag{9}$$

In the case of this simple linear regression, y is not only expected to equal β_1 times a variable (x) plus a constant, but is also dependent on an error term, ε. The convention is to assume that this error term is normally distributed with a mean of zero and constant standard deviation $[\varepsilon \sim N\ (\mu = 0), \sigma]$. This assumption of normality with a constant standard deviation in the error term is what is meant by homoscedasticity in that it is not dependent on the size of x. Later, we will define heteroskedasticity and determine how to deal with the problems it poses to our volatility forecasts.

When dealing with a time series, we refer to μ_t and σ_t^2 as the unconditional mean and variance of the series. A different way to think of the unconditional mean and variance of a time series is as a long run average and variance of the series, with no additional weight placed on more current information. However, since we wish use both past and current information to make forecasts about the future, it is helpful to use conditional means and variances where our forecast is dependent on all information known at time t. If we consider a distribution of a variable x, for example, the variable $x_{t=2}$ is dependent on, or conditional on, the information garnered from $x_{t=1}$. Therefore, using the information at hand at time $t - 1$, we can define the conditional mean and variance as $\mu_t | X_{t-1}$ and $\sigma^2 | X_{t-1}$.[19] In terms model, we can predict X_{t+1} in an autoregressive fashion by regressing it against current and past values of the same variable as follows:

$$X_{t+1} = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + \cdots + \beta_n X_{t-n} + \varepsilon_{t+1} \tag{10}$$

[19] Engle et al., "ARCH/GARCH Models in Applied Financial Econometrics," 6.

When looking at ARCH models, the emphasis is on the error term. The significant breakthrough provided by ARCH is the realization that we can create a stochastic process for the error terms that can predict their average size when fitted to data. Having built up to this, we can now move on to studying these more complicated models in further detail and gain an understanding for how they can improve volatility forecasts beyond the methods considered up until now.

2.4 – ARCH & GARCH

In order to fully comprehend the ARCH class of models, we must understand the concept of heteroskedasticity. What exactly does it mean for a set of data points to be heteroskedastic? Consider the following cross sectional regression analysis where we try to predict annual income from a person's age. Since the majority of teenagers aren't wealthy compared to older people and since incomes vary widely among adults, one could imagine that a scatterplot of the two variables—with age on the horizontal axis and annual income on the vertical axis—would look somewhat like a megaphone. In other words, there are certain subsets of the larger set that have different means and variances. In finance, when we look at a collection of random variables, such as a time series of returns, we often see the same thing. There are certain subsets of time that will exhibit returns with different means and variances from the rest of the series. Specifically, asset returns tend to exhibit clustering effects where increases in variance are highly correlated with further increases in variance, and vice versa.[20] There are several reasons as to why this is the case. Consider, for example, the downside portfolio protection actions taken by

[20] Mandelbrot, B.B. "The Variation of Certain Speculative Prices." *Journal of Business* 36.4 (1963): 418.

long only portfolio managers. If equities exhibit a substantial drop, automated risk management systems or emotional selling could contribute to subsequent drops in equities. Additionally, since many portfolio managers are already highly correlated with their benchmark index, the downward volatility could increase further. This is the 'conditional' component to the ARCH class of models. These significant drops can lead to variances that are highly correlated with each other and therefore are dependent—or conditional—on all the available information. The ARCH class of models elegantly captures this tendency.

To further study this tendency, let us consider the following autoregressive model, where we try to predict returns from the previous period's return, a constant, and an error term.

$$r_t = \kappa + r_{t-1} + \varepsilon_t \tag{11}$$

where r_t is the return on an asset or portfolio at time t. In an ARCH(1) model, where we are concerned with the error term, we write this term as

$$\varepsilon_t = \sigma_t w_t \tag{12}$$

and therefore,

$$r_t = \kappa + r_{t-1} + \sigma_t w_t \tag{13}$$

where w_t is discrete white noise distributed normally with a mean of zero and unit variance and σ^2 is a constant plus some multiple of the squared residual in the previous period. Substituting for σ^2, we get:[21]

$$\varepsilon_t = w_t\sqrt{\alpha_0 + \alpha_1\varepsilon_{t-1}^2} \tag{14}$$

[21] Enders, Walter. *Applied Econometric Time Series* (Hoboken: John Wiley & Sons, 2004), 114.

ARCH becomes a forecasting model in the sense that it predicts the variability of the errors at time t based on the information known at time t – 1. Additionally, having information on the past errors means that ARCH models do not create any doubts on the expectation of the squared errors at time t. Both of these conditions are necessary for a forecasting model even if, as in the case of the second condition, the squared errors can hypothetically diverge widely from the forecast. If we want to extend the ARCH(1) model to one that can accommodate for higher order lags, we would denote it as an ARCH(p) process.[22]

$$\varepsilon_t = w_t \sqrt{\alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2} \tag{15}$$

Similarly, we can denote a GARCH(p, q) model of order p, q as

$$\varepsilon_t = \sigma_t w_t \tag{16}$$

where

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{\square} \beta_j \, \sigma_{t-j}^2. \tag{17}$$

where ω is a parameter of the model.[23] This is similar to an ARCH(p) model except that we add on the moving average, or GARCH, terms. The p refers to the number of autoregressive lags implemented for the ARCH terms, while the q refers to the number of moving average lags or GARCH terms. For example, a GARCH(1,1) model would specify one lag for the ARCH terms and one lag for the GARCH terms. Essentially, a GARCH model predicts variance in the next period using a weighted average of past squared residuals with declining weights that never go to zero. The weights are the long-

[22] Enders, Walter. *Applied Econometric Time Series* (Hoboken: John Wiley & Sons, 2004), 114.
[23] Enders, Walter. *Applied Econometric Time Series* (Hoboken: John Wiley & Sons, 2004), 118.

run variance ($\omega = 1 - \alpha - \beta$, a constant), the predicted variance for the current period (β), and the new information gained in the current period based the most recent squared residual (α). Therefore, a GARCH(1,1) model for variance could be written succinctly as:[24]

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{18}$$

Finally, we want to look at the concept of volatility persistence, which shows how long it takes for volatility to revert towards its "normal"—or long run—level. This can be done as follows:

$$\alpha_t + \beta_t < 1 \tag{19}$$

where $\alpha_t + \beta_t = 1$ implies that the current period is exhibiting volatility persistence, meaning that it continues to stay high and displays no mean reverting tendencies.[25] It should also be noted that $\alpha + \beta$ must be less than or equal to one otherwise the series becomes unstable and that the model makes the most intuitive sense when the weights are all positive.[26]

2.5 – Some Background Literature

Ever since Engle's (1982) seminal article introducing the ARCH model and Bollerslev's (1986) generalized version (GARCH), thousands of articles have been written on the subject trying to capture the inherent volatility clustering found in financial time series data. Mandelbrot (1963) originally introduced the concept of volatility

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[24] Enders, Walter. *Applied Econometric Time Series* (Hoboken: John Wiley & Sons, 2004), 118.
[25] Enders, Walter. *Applied Econometric Time Series* (Hoboken: John Wiley & Sons, 2004), 133.
[26] Engle et al., "ARCH/GARCH Models in Applied Financial Econometrics," 8.

clustering or persistence, which he described as "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes."

There have also been some interesting and important extensions of the GARCH class of models. Nelson (1991) realized that GARCH had three distinct disadvantages. First of all, he points out that current returns are negatively correlated with future returns volatility. Secondly, some of the parameters in GARCH are restricted to the point where they limit the effect of the conditional variance process. And finally, he argues that it is difficult to determine whether or not volatility shocks persist or not with GARCH. Therefore, Nelson proposes an exponential ARCH that accommodates asymmetric conditional volatility from negative, as opposed to positive, shocks. Glosten et al., (1993) use an altered GARCH-in-mean (GARCH-M) model that allows for seasonality as well as asymmetry in conditional variance. Bekaert et al. (2015) develop an extension of the asymmetric GARCH framework, which they coin "bad environment, good environment" or BEGE. Using monthly data for the U.S. stock markets from 1929 through 2010, they develop their model such that it accommodates two shock parameters—a good shock and a bad shock. Furthermore, the model accommodates for the tendency of bad shocks to have a greater effect on the lower end of the returns distribution than good shocks.

There have been some mixed positions on the effectiveness of GARCH as a forecasting tool. Day and Lewis (1992) look at weekly returns for the S&P 100 index from 1983-1989 using a GARCH (1,1) and report an R^2 = 0.039. Using a GARCH (1, 2) model for monthly returns of the U.S. stock market from 1835 to 1925, Schwert (1990) reports R^2 = 0.067. Even Andersen and Bollerslev (1998) report R^2's of 0.047 and 0.026

for the Deutschemark-dollar and yen-dollar exchange rates. However, they argue that using R^2 as a measure of accuracy for forecasting volatility is troublesome given that forecasting depends on expected future volatility as opposed to the prior realized squared returns. They show that the anticipation of a traditional high R^2 is incompatible with these types of volatility models. Based on this, they conclude that, contrary to popular belief, the ARCH class of models actually do provide accurate forecasts of future volatility.

Others will disagree, however. Choudhry and Wu (2008) look at the forecasting ability of four different GARCH models compared to the Kalman filter method. Using daily stock returns for 20 UK companies, they find that a GARCH-GJR model provided more accurate forecasts than the other GARCH models, but all performed worse than the Kalman filter method. Hanson and Lunde (2005) look at 330 ARCH class models and find that a simple GARCH (1, 1) model is not outperformed by any of the other more sophisticated models when looking at out of sample deutschemark-dollar exchange rate data. However, it was outperformed and performed poorly overall when looking at IBM returns, due to its failure to incorporate a leverage effect. Finally, Jorion (1995) finds that GARCH models are outperformed by implied standard deviation (ISD) models when looking at daily data on options in the foreign exchange market.

3. Diversification

Section two looked at some of the different methodologies academics and market participants utilize to measure risk and forecast volatility while also developing one of the most prominent models for forecasting volatility. Having examined some of the building blocks of risk and traced the evolution of the GARCH class of models, we can now look at how to apply these models to learn more about international diversification. Particularly, the home bias theory is introduced to give some background on how to apply Bollerslev's model in an empirical way. After introducing the home bias theory and providing some potential theories for the existence of this occurrence, a new method for managing risk is proposed. Namely, instead of achieving international diversification through owing a higher percentage of international equity, can investors diversify internationally by owning U.S. based companies that are themselves diversified? If so, then this strategy could potentially offer investors some of the benefits of diversification while offering a more psychologically appealing investment. That is, by owning domestic stocks, which past research shows is preferred by most investors, can investors potentially diversify away some of the risk that comes with having holdings that are so concentrated in a single economy? Ultimately, section four will help provide answers to some of these questions. Before arriving there, however, the home bias puzzle is introduced, company and index portfolios are created and explained in further detail, and the research question of how diversified companies can help reduce an investor's overall risk profile is put forward.

3.1 – The Home Bias Enigma

One of the more neglected areas where modeling risk might be useful deals with the theory of home bias. The home bias enigma describes the reality that the majority of individuals in numerous countries invest primarily in domestic securities, despite the well-documented benefits of internationally diversified portfolios. In the majority of cases, a person's income is already closely tied to the performance of the economy in their home country so the basic principles of finance tell us that they should be diversifying out of their country of residence. However, according to Vanguard, U.S. equities represented nearly half of the global equity markets as of December 31, 2013. Yet despite this, U.S. mutual fund investors only held 27% of their total equity allocation in foreign equities on average, up from 1% in the mid 1980s and 12% a decade ago.[27] Even so, this allocation is low compared to the share of foreign equities in the global market.

This raises the question of whether or not there is a good explanation for this behavior, especially since modern portfolio theory and the international version of the capital asset pricing model (CAPM) tells us to hold a diversified global market portfolio of risky assets. Arguments for barriers to international investment no longer provide a suitable explanation for the home bias puzzle given the recent increases in popularity of exchange traded funds (ETFs) and mutual funds that provide international exposure. Since investors have increased access to international markets, perhaps there is a different explanation for their reluctance to invest away from home. It is possible that bias is too unfair of a word and investors actually make rational decisions in choosing to allocate such a high percentage of their equity exposure to the U.S. markets.

3.2 – Background Literature on International Diversification

Many authors have written on the benefits of international diversification. Grubel (1968) expanded on Markowitz's model of portfolio balance to show how international

[27] Vanguard, "Global Equities: Balancing Home Bias," New York, 2013.

diversification leads to a completely new source of gains for investors. Markides and Ittner (1994) found that companies that acquired international companies showed a positive benefit to the shareholders of the acquiring company. De Santis and Gerard (1997) apply GARCH to the CAPM. They find that U.S. market downturns can affect the international markets and therefore erode some of the benefits of holding an internationally diversified portfolio. Despite this, they estimate that the gain from diversifying internationally is an additional 2.11 percent annually. Furthermore, this return had not been affected by growing integration between markets and economies around the world.

Others propose potential explanations for the existence of the home bias puzzle. Merton (1987) and others suggest that indirect barriers exist that prevent investors from increasing their foreign equity exposure. Specifically, information costs, such as investors' higher perceived riskiness of companies they don't know and willingness to invest in companies they do know, affect portfolio construction. Coval and Moskowitz (1999) suggest that investors have a tendency to invest in companies that are physically closer to them. They found that the average U.S. fund manager invests in companies that were nine to eleven percent closer than firms they could have held. Therefore, they conclude that information asymmetry can play a major role in investment decisions and portfolio construction. Ahearne et al. (2004) propose a different type of role when it comes to information—regulation. For U.S. investors, if a foreign company is listed on a U.S. exchange and therefore, has to adhere to the rules of the U.S. regulatory environment, then the information costs associated with investing in such a company fall dramatically. They suggest that if all foreign firms were listed in the United States, and foreign markets kept their 50 percent share of the global equity market, the share of foreign equities in a U.S. portfolio would increase from 10 percent to 25 percent on average, at the time of writing. Van Nieuwerburgh and Veldkamp (2009) look at why the widespread reach of information many people enjoy in today's world does not eliminate information asymmetry. They look at the learning capabilities of investors and find that even if an investor can choose to learn what foreigners know, they choose not to. This only amplifies information asymmetry between foreign and domestic investors.

3.3 – Data Description

This thesis examines how risk changes based on the underlying type of company and based on the type of market to see if there are rational reasons for investors' home bias tendencies and to see if there are alternatives to mutual funds and exchange traded funds that can offer investors international exposure. Specifically, local companies that are themselves internationally diversified are analyzed to see if they can provide a psychologically more appealing alternative to mutual funds, ETFs, or foreign securities. This process begins broadly by looking at the performance and risk characteristics of some major stock indices around the world and later moving on to examine the characteristics of United States companies that meet a certain criteria. Looking at indices is done to determine if certain they can provide increased diversification benefits when compared to the two company based portfolios that will be described in further detail later on. Specifically, the Standard and Poor's 500 (S&P 500), Financial Times Stock Exchange 100 (FTSE 100), Deutscher Aktienindex (DAX), Nikkei 225, and MSCI All

Country World Index (MSCI ACWI) are looked at. The S&P 500 is made up of the 500 largest companies listed on U.S. exchanges and is widely followed by investors and traders around the world. Similarly, the FTSE 100 is an index of the 100 largest companies listed on the London Stock Exchange. The DAX is an index comprised of the 30 largest German companies traded on the Frankfurt Stock Exchange by market capitalization and trade volume. The Nikkei 225 is a price-weighted index that tracks companies listed on the Tokyo Stock Exchange. Finally, the MSCI ACWI covers the majority (85%) of the world's potential equity investments by tracking mid and large cap companies in 23 developed market countries and 23 emerging market countries.[28]

After analyzing the broader indices to see if there are any explicit benefits to holding one index over any of the others, an analysis of individual companies that are headquartered in the United States is conducted and looks specifically at whether or not a company that is itself internationally diversified can provide investors with additional diversification benefits. To this end, two portfolios are constructed—one with companies that obtained the majority of their revenue (>50%) from the United States and the other that gained a significant portion of their revenue from outside of the United States (>40%)—over the period beginning January 1, 2005 and ending December 31, 2014. Each 'domestic' company is matched with a comparable 'international' company in terms of sector and size, with only a few exceptions. Collectively, the companies span a variety of sectors and subsectors and have different market capitalizations so as to create a diversified portfolio by size and sector within each category. A complete listing of the

[28] MSCI, "MSCI ACWI (USD)," New York, 2016.

companies in each category can be found below in table 1. Once the companies were selected, two equally weighted portfolios were created. The performance and risk metrics of these two portfolios are further analyzed in section 4.

Table 1: Domestic & International Portfolio Components

Domestic Company	**Market Capitalization**[*+]	**Sector**	**International Company**	**Market Capitalization**[*+]
Altria Group	113.39	Tobacco & Cigarettes	Universal Corporation	1.15
American Express	70.60	Credit Services	MasterCard	110.55
Biomarin	16.79	Biotechnology	Illumina	27.35
Bristol Myers	111.83	Drug Manufacturers	Eli Lilly	91.12
Cisco	139.24	Technology	HP Inc.	22.95
Devon Energy	13.09	Energy	Exxon Mobile	339.74
Halliburton	34.28	Energy	Schlumberger	97.53
Hershey	18.68	Consumer	Mondelez	70.30
Monsanto	42.77	Agricultural Chemicals	Dow Chemical	61.28
Oracle	152.71	Technology	Intel	165.14
Simon Property Group	57.98	Property	CBRE Group	12.50
UnitedHealth	110.34	Healthcare	Abbott Labs	67.45

[*]As of 12/262015

[+]In Billions of U.S. Dollars

Finally, a GARCH(1,1) model is applied to the S&P 500, MSCI ACWI, Domestic, and International portfolios. The model is applied over four periods—the whole period of the data set beginning January 1, 2005 and ending December 31, 2014, the pre-financial crisis period (January 1, 2005 – March 31, 2007), the financial crisis period (April 1, 2007 – March 31, 2009), and the post-crisis period (April 1, 2009 – December 31, 2014). Doing so gives the added benefit of analyzing how volatility changes over time and during periods of extreme market stress. There are two expectations with regards to the volatility of each time period. First, one would expect that, during the financial crisis, the previous day's volatility would affect the current day's volatility a lot more than during the pre or post crisis periods. Additionally, the persistence of volatility should be higher during the crisis indicating heightened risk aversion amongst investors and traders. The second expectation is that the structure of volatility changed following the crisis compared to what it was pre-crisis. Given the fact that investors had just endured one of the worst financial crises in history, it is not unreasonable to presume that their investment decisions were partially influenced by the events of a few years prior. It is likely that bad news in the post crisis period led to a greater overreaction than it would have in the pre-crisis period, which would mean that the volatility in prior periods would have an even greater influence on the current period.

When analyzing the performance and risk characteristics of the two company portfolios and two index based portfolios, there are several outcomes that could occur. With regards to the company portfolios, the primary hypothesis is that the difference between the risk characteristics of each basket of companies is equal to zero. This would mean that there was no difference between the risk characteristics of each portfolio and any statistically significant rejection of this hypothesis would indicate that one of the portfolios exhibited more preferable risk characteristics than the other. If this were the case, the expected result is that the International portfolio should perform better since it offers more diversification benefits than the Domestic portfolio. If this were true, then there exists the potential to manage risk by investing in U.S. companies that generate significant portions of their revenue from overseas. Given that the companies in the international portfolio are collectively exposed to numerous economies around the world and that the performance of those in the domestic portfolio are tied largely to the production of the U.S. economy, a clear case could be made for the outperformance of the International portfolio based on some of the fundamental principles of finance. On the other hand, one could argue that, since the U.S. economy is the largest and one of the strongest in the world, companies with significant exposure to the U.S. would have performed better, especially during the crisis and ensuing recovery. When comparing the two index portfolios, one would again hypothesize that there was no difference between their risk characteristics and any rejection of this hypothesis would show that either a diversified international portfolio or a domestically heavy portfolio is preferable with regards to their risk characteristics. Similar to the company portfolios, one could argue that that the MSCI ACWI should outperform the S&P 500 given that it spans the stock markets of over 20 different countries. Therefore, the diversification benefits should cause it to outperform. However, it is also possible that the S&P 500 performed better due to the strength of the U.S. economy. All of these scenarios will be further analyzed in section 4.

4. Modeling Risk and Diversification

4.1 – Performance Statistics

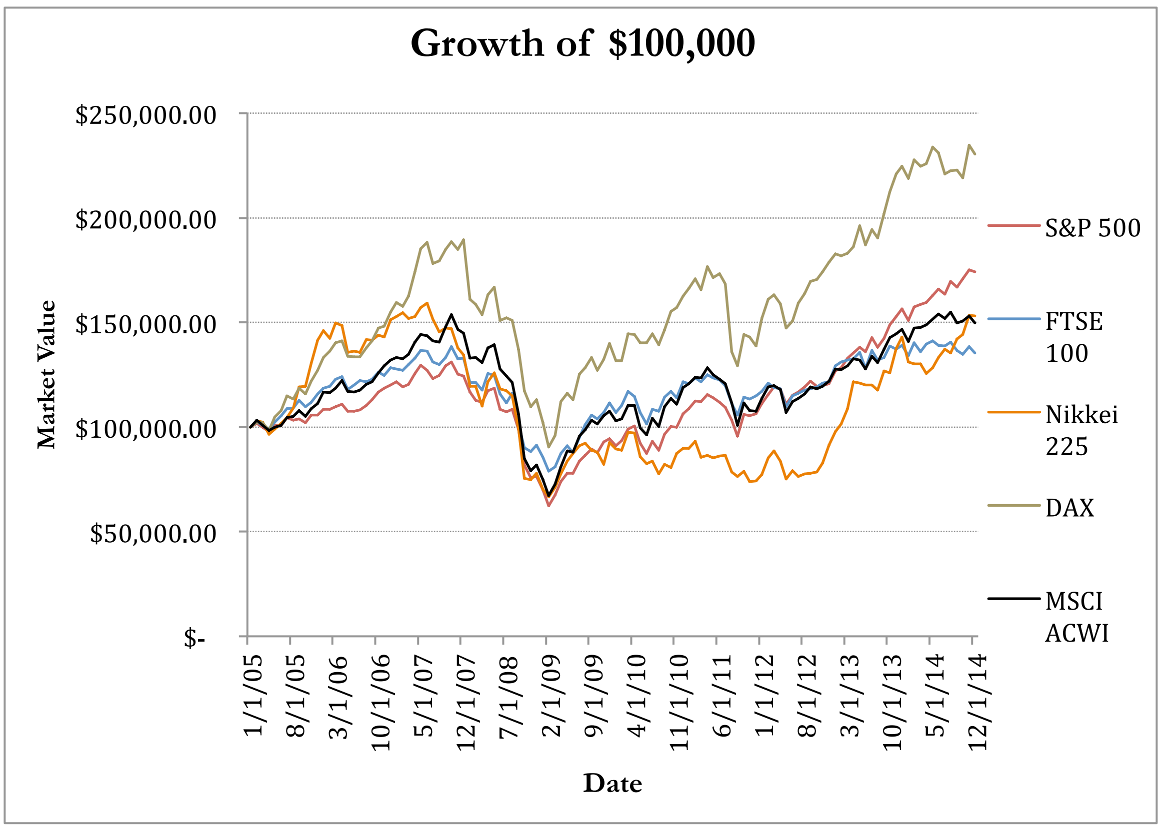
When looking at the performance of the index and company portfolios described in section 3 in order to study the impact that a globally diversified portfolio has on risk, it is important to look at both the overall performance and whether or not that performance was achieved by taking on excessive amounts of risk. Figure 2 gives us a look at the growth of $100,000 invested in each of the aforementioned indices, excluding dividends. On an absolute return basis, Germany's DAX outperformed the other indices, while the MSCI ACWI underperformed in many cases. Depending on which measure you look at and an investor's investment philosophy, the interpretation of the results could vary, however. For example, an investor that is not capable of stomaching the volatility associated with higher returns might prefer a portfolio made up of the S&P 500 over one comprised of the DAX, since the standard deviation of the returns was lower over the timespan of the data set. This means that the swings in the portfolio value were not as drastic and therefore might put an investor's mind at ease, which can be and important factor when investing.

Another way to compare the portfolios could look at what the maximum loss was from the portfolio's prior peak to ensuing trough, which is defined as the maximum drawdown on the portfolio. Even though the S&P 500 and Nikkei 225 are nearly similar in terms of risk-adjusted returns, for example, the maximum drawdown of the S&P 500 was over $30,000 less than the Nikkei's. Again, for an investor who is less willing to stomach large bouts of volatility, a lower maximum drawdown could sway their preference for a given security or portfolio. A complete summary of the performance statistics for all the portfolios can be found in table 2.

Table 2: Performance Statistics of Different Portfolios

Portfolio	Annualized Return	Standard Deviation	Sharpe Ratio	Maximum Drawdown
		Full Period		
Domestic	18.83%	20.03%	0.78	$102,012.78
International	19.12%	22.17%	0.72	$101,835.13
S&P 500	5.72%	14.96%	0.20	$68,933.43
FTSE 100	3.07%	13.94%	0.03	$59,590.30
DAX	8.71%	18.60%	0.35	$92,819.81
Nikkei 225	4.36%	20.61%	0.17	$99,265.07
MSCI AWCI	4.13%	16.94%	0.07	$86,403.16
		Pre-Crisis Period		
Domestic	25.25%	13.52%	1.64	--
International	25.93%	12.98%	1.76	--
S&P 500	7.77%	10.37%	0.44	--
MSCI ACWI	13.49%	9.30%	1.13	--
		Crisis Period		
Domestic	(14.99%)	37.63%	-0.68	--
International	(8.43%)	34.19%	-0.55	--
S&P 500	(25.09%)	34.14%	-1.04	--
MSCI ACWI	(23.22%)	26.97%	-1.25	--
		Post-Crisis Period		
Domestic	26.71%	17.58%	1.39	--
International	23.71%	17.58%	1.22	--
S&P 500	17.92%	16.67%	0.93	--
MSCI ACWI	13.90%	14.83%	0.43	--

Similarly, we can compare the two company based portfolios. Figure 3 shows the growth of $100,000 in each of those portfolios. There are few discernable differences between the two portfolios at first glance as both portfolios outperformed the other at certain times. Ultimately, the international portfolio outperformed the domestic portfolio on a total return and lower maximum drawdown basis, although the domestic portfolio achieved the better risk adjusted return over the period. The following subsection will take a deeper look into the performance of these two portfolios by using Bollerslev's GARCH(1,1) model. This will provide further clarity on the risk characteristics of each portfolio and, based on the conditional variances of each, will help decide whether or not international diversification can be achieved while still owing U.S. headquartered companies.



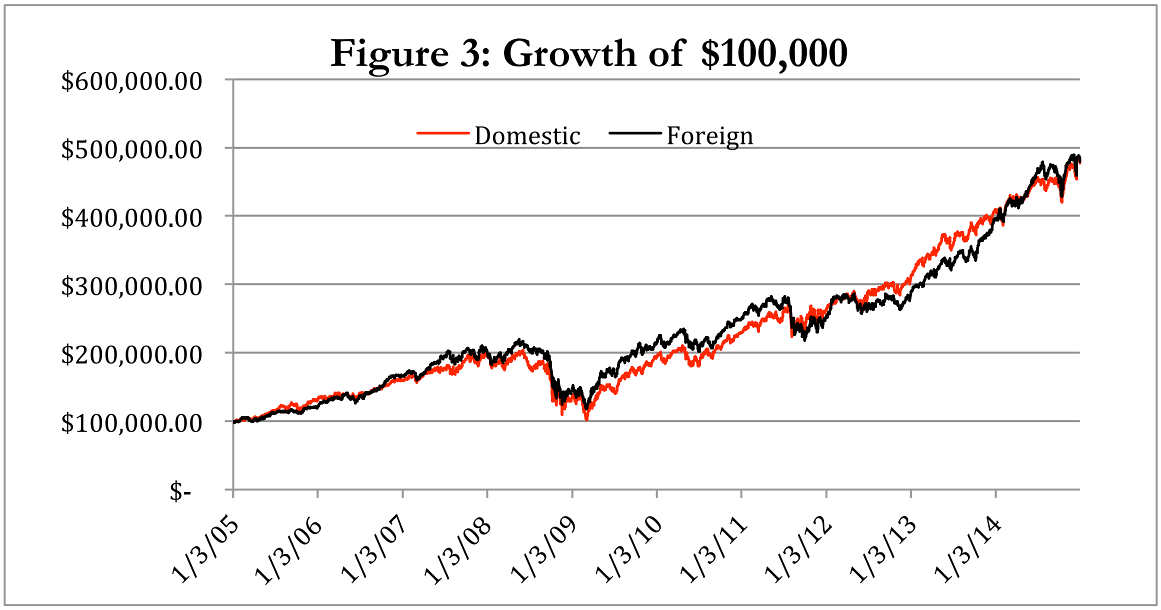


Figure 4 shows the daily returns of both the domestic and international portfolios. Looking at a graph like this can help give a sense of the volatility each portfolio endured

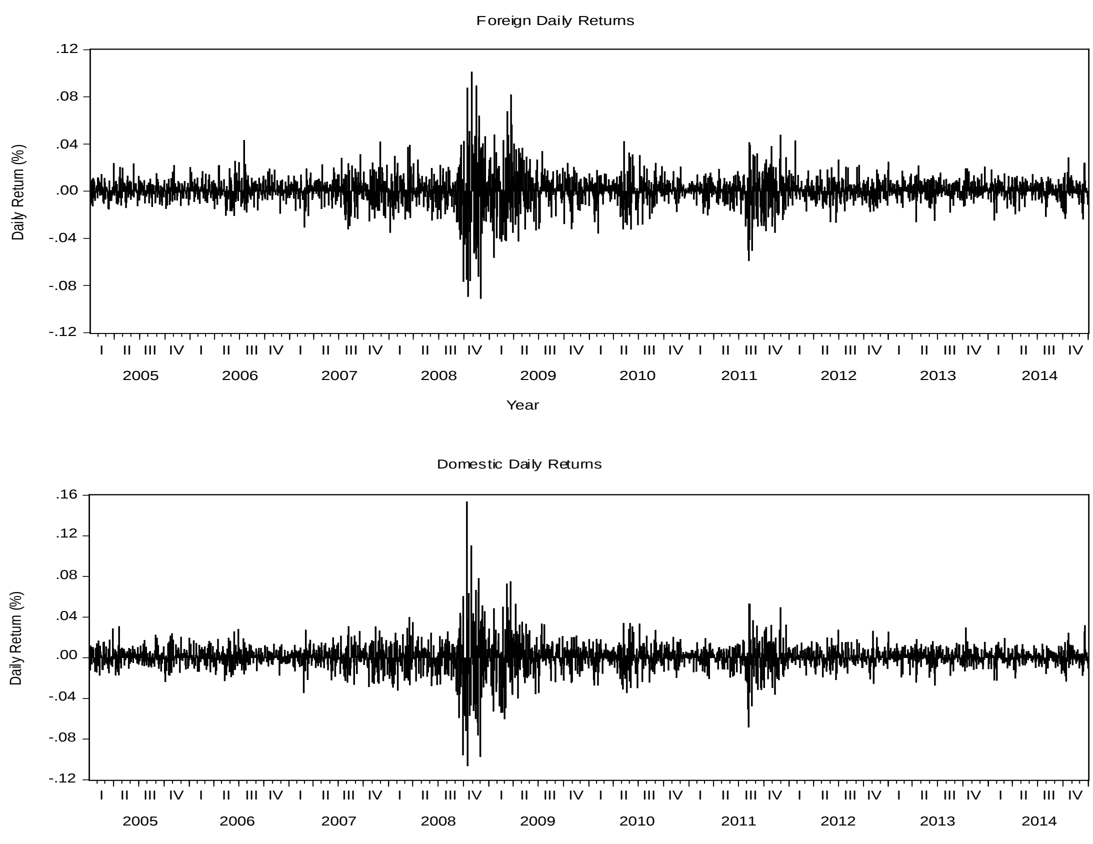


Figure 4: Data is the daily log returns of both the domestic and foreign portfolios. The domestic portfolio is on the top panel while the foreign portfolio is on the lower panel.

throughout the span of this analysis. Although there are few differences between the two portfolios, it should be of note that the volatility for both portfolios increased drastically during the financial crisis. Strangely, the upward volatility for the domestic portfolio seemed higher than the downside volatility. Another noticeable difference comes in the latter half of 2011, where it appears as if the international portfolio experienced a slightly more volatile spell than the domestic portfolio. Perhaps this is due to the various concerns about the health of different European governments that were sweeping across Europe at the time.

4.2 – Model Specification & Results

Using Bollerslev's GARCH(1,1) model, shown previously in equations (11) and (18), the S&P 500, MSCI ACWI, Domestic, and International portfolios were compared across the different four different time periods described in section 3.3. The results of this model can be found in the tables in Appendix B. Across all portfolios the previous day's return was statistically significant at both the five and ten percent significance levels during the financial crisis, whereas it had not been significant both prior to and after the crisis in three out of four cases. This indicates, perhaps, that volatility persistence must have been higher during the crisis than both before and after. The results of the GARCH model confirm this. During the financial crisis, $\alpha_{t-1} + \beta_{t-1}$ gets very close to 1 in all cases, indicating that volatility did not exhibit any mean reverting tendencies. Instead, volatility remained unusually high compared to pre-crisis levels. This is not unexpected as, during this period, investors and traders were consistently fearful of further developments that could threaten the survival of the financial sector. This concern is echoed by the jump in the coefficient for the ARCH term from pre-crisis to crisis. The ARCH terms were fairly low across the board for the pre-crisis period indicating that the previous day's volatility did not matter as much in the current period. This all changes during the financial crisis, however. During the crisis, the ARCH coefficients rise showing that the previous day's volatility was much more important to the current period's volatility estimate than before. This is expected as times of extreme market stress—as was the case in the 2007 to 2009 period—bring about great uncertainty about the future, which leads to intensified risk aversion among investors and traders. A final notable fact about the ARCH term is that it did not decrease significantly following the crisis and certainly did not revert to pre-crisis levels. The fact that the ARCH term remained elevated after the crisis could show that investors still had the events of the financial crisis on their minds and that any signs of market turbulence brought about a greater reaction than they would have before the crisis.

Figure 5—located in appendix A—shows the forecasts provided by the GARCH(1,1) model when applied to each of the four portfolios. Similar to the VIX, shown in figure 1, the conditional variance begins to increase in mid to late 2007 before exploding in the second half of 2008. The conditional variance did not return to pre-crisis levels until the back half of 2009, which coincides with the beginning of the current bull market. Taking a closer look at the conditional variances for each portfolio provides some evidence for the outperformance of the foreign portfolio. During the financial crisis, the conditional variance for the domestic portfolio is nearly double that of the foreign portfolio indicating that the volatility of the domestic portfolio was higher than that of the

foreign portfolio. Wilcoxon tests of median equality were performed and also confirmed this fact. The conditional variance of the domestic portfolio was higher than that of the foreign portfolio at the one percent significance level for both the entire period and the crisis period indicating that the risk characteristics of the foreign portfolio were preferable during these time periods. The results of the Wilcoxon test indicate that we can reject the primary hypothesis posed in section 3 for the entire time period and for the crisis period. This shows that there are potential diversification benefits to be gained from investing in a portfolio of U.S. headquartered companies that are themselves internationally diversified, especially during times of extreme market stress. Additionally, as mentioned earlier, this could benefit certain investors who are not able to stomach large bouts of market volatility when the markets become turbulent. Combined with the performance statistics described in the prior subsection, there is evidence that adding internationally diversified companies to a portfolio can help manage the risk of that portfolio, all while providing a psychological advantage to investors who dislike owning foreign equities. A noteworthy caveat must be made about these findings, however. Interestingly, the conditional variance of the international portfolio was statistically significantly higher than the domestic portfolio in the post crisis period. This shows that perhaps the U.S. markets were the best place to invest following the crisis given the relative strength of the U.S. economy during the recovery period. It is also possible that this resulted from a number of the companies in the foreign portfolio having exposure to Europe following the financial crisis. Of course, Europe had a difficult few years directly following the crisis as their overleveraged governments tried to deleverage and austerity measures crippled economic growth.

The foreign portfolio also displayed a lower conditional variance than the S&P 500 during the financial crisis, although it was not statistically significant suggesting that a portfolio made up solely of these types of companies is not necessarily a better alternative than investing in the entire market index, especially during times of crisis. However, the MSCI ACWI did display a statistically significantly lower conditional variance than the S&P 500 both during and following the crisis. This is noteworthy given how poorly this index performed on the other performance and risk metrics outlined in the prior subsection. Although the index performed poorly on some of those other performance and risk measures, the MSCI ACWI's outperformance as specified by the GARCH(1,1) model provides further evidence in favor of international diversification, particularly during severe downturns in the market. This fact also helps promote the idea that portfolios similar to the foreign portfolio constructed in this thesis could help investors better manage risk during times of market upheaval. Although the foreign portfolio performed in line with the domestic portfolio when looking at the risk and performance metrics in the prior subsection, the GARCH(1,1) model seems to indicate that portfolios similar to the foreign portfolio can provide additional benefits during financial crises that could appeal to many investors. According to the findings in this thesis, investing in portfolios similar to the foreign portfolio constructed here can achieve similar upside to U.S. companies with heavy domestic exposure, all while sustaining more preferable risk characteristics during times of global financial turbulence. This, combined with the added evidence provided by the lower volatility exhibited by the MSCI ACWI, points to a compelling case in favor of U.S. companies that generate significant revenues from outside the U.S. A complete summary of the Wilcoxon tests can be found in table 3 below.

Table 3: Wilcoxon Tests for Select Pairings

Portfolio Pairing	**Test Statistic**	**Probability**
	Full Period	
Domestic – Foreign	5.299	< 0.0005
	Crisis Period	
Domestic – Foreign	10.492	< 0.0005
S&P – MSCI	19.33	< 0.0005
S&P – Foreign	0.960	0.337
S&P – Domestic	-14.262	< 0.0005
MSCI – Foreign	18.234	< 0.0005
	Post Crisis Period	
Domestic – Foreign	-3.789	< 0.0005
S&P – Foreign	-21.019	< 0.0005
S&P – Domestic	-21.051	< 0.0005
S&P – MSCI	22.932	< 0.0005

5. Conclusion

This thesis studied some of the different measures academics and market participants use to quantify risk and forecast volatility. Additionally, this thesis thoroughly analyzed four different portfolios made up of the S&P 500, MSCI ACWI, and two different company portfolios that were constructed based on where different U.S. headquartered companies generated the most revenue. The overall goal was to determine if a portfolio comprised of U.S. based companies that received a large part of their sales from overseas could provide significant international diversification benefits to investors and traders so as to increase their international exposure while still providing the psychological comfort that comes with owning a domestic security. Furthermore, times of financial crisis were analyzed to see how risk characteristics change during market turmoil and to see what additional benefits could come from holding a diversified portfolio during these tumultuous times. Although the portfolio comprised of companies that earned a large part of their revenue outside the United States performed in line with the portfolio made up of companies that earned the majority of their revenue in the United States with regards to measures of overall return, standard deviation, Sharpe ratios, and maximum drawdown, the evidence provided by a GARCH(1,1) model suggests that the foreign portfolio exhibited lower conditional volatility throughout the study and especially during the financial crisis. This indicates, perhaps, that there are benefits to holding a basket of companies that are themselves internationally diversified. Additionally, the MSCI ACWI exhibited lower conditional volatility than all the other portfolios throughout all the different time periods analyzed, providing evidence in favor of investing internationally in order to help mitigate investment risk. Of course, as indicated in section 3, this is not always preferable to the majority of investors. Therefore, combined with the strong performance of the foreign portfolio, this presents evidence that a portfolio comprised of U.S. headquartered companies that are themselves internationally diversified can give investors the ability to better manage investment risk using investment vehicles that are more psychologically comforting than anything else currently available.

Further research could take this process a step further in a number of ways. First and foremost, future studies could focus in on different regions of the world to determine if that plays a role in the performance of a portfolio. For example, it was hypothesized in section 4 that the underperformance of the foreign portfolio following the financial crisis was partly due to the austerity related problems that were taking place in Europe at the time. Of course, this presumption was not proved in this thesis so future studies could look at foreign portfolios of U.S. companies that have significant exposure to Europe to see if that was in fact the reason for the underperformance. Future studies could also analyze more companies and rank them based on how much exposure they have to international economies. Doing so could help solidify the research undertaken in this thesis and help ensure that U.S. headquartered companies with international exposure can help manage investment risk by providing psychologically more appealing investments that offer international diversification. Finally, future research could also apply a similar framework as the one employed in this thesis to see how risk characteristics change by both industry and size as perhaps international diversification carries greater benefits depending on the size and industry of the company. Additionally, given the prevalence of upside volatility displayed by the domestic portfolio during the financial crisis, future research could apply an asymmetric model to similar types of portfolios to account for the existence of both opportunity and danger in the financial markets. This type of framework is beyond the scope of this thesis so it is left to further research.

Appendix A: GARCH Predictability

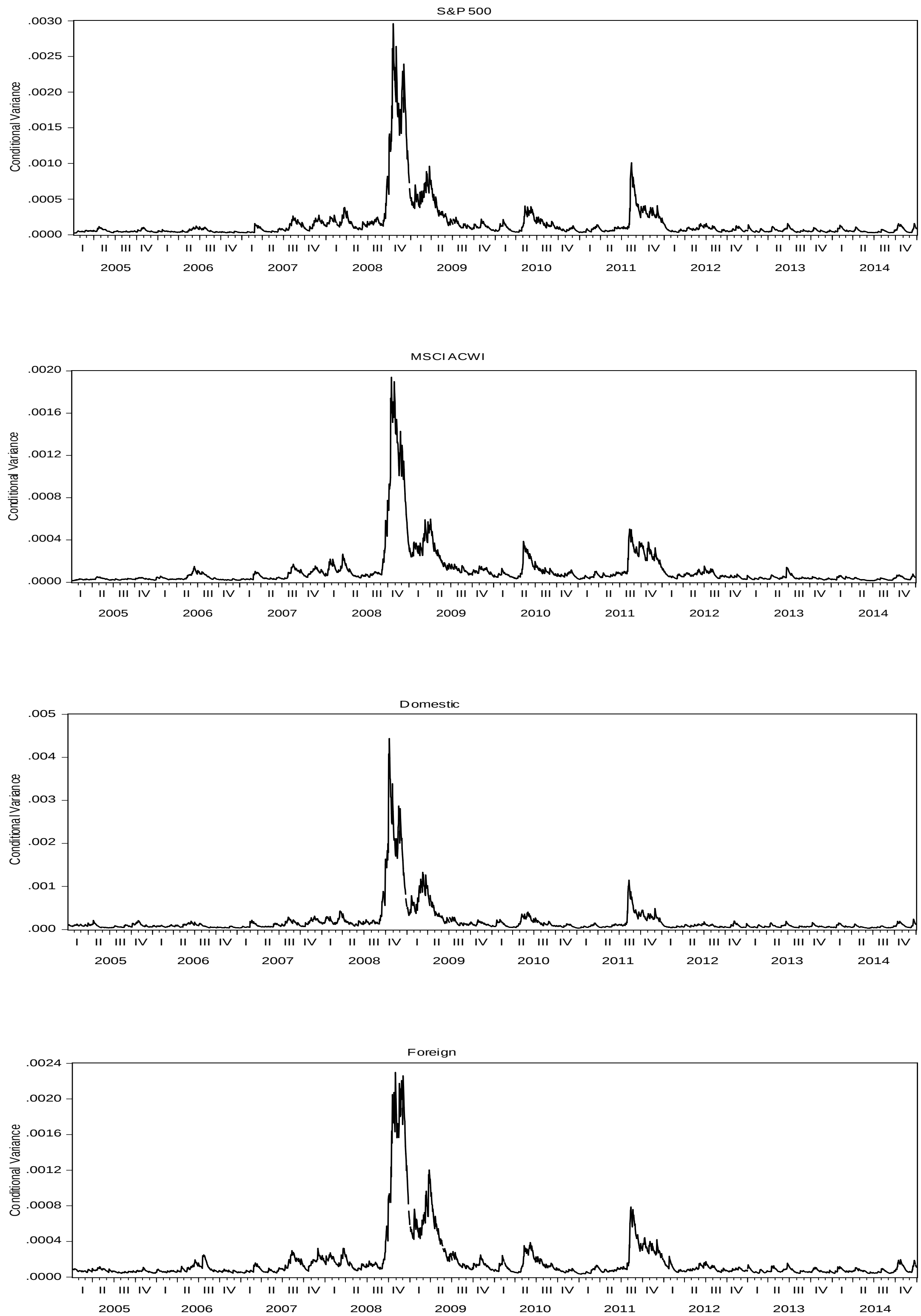


Figure 5: The estimation of the GARCH(1,1) model shows the conditional variances for the four different portfolios. Data is derived from daily log returns from January 1, 2005 to December 31, 2014. Source: Yahoo! Finance.

Appendix B: Results of a GARCH(1,1) Estimation

S&P 500: Full Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.000667	0.000165	4.051	0.000
R_{t-1}	-0.066948	0.023068	-2.90213	0.004
Variance Equation				
ω	1.93E-06	2.93E-07	6.5920	0.000
α_{t-1}	0.100159	0.009437	10.614	10.614
β_{t-1}	0.883864	0.010606	83.339	0.000

MSCI ACWI: Full Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.000468	0.000136	3.441326	0.001
R_{t-1}	0.157145	0.021105	7.445689	0.000
Variance Equation				
ω	7.76E-07	1.64E-07	4.733696	0.000
α_{t-1}	0.083266	0.007583	10.98086	0.000
β_{t-1}	0.909511	0.007979	113.9883	0.000

S&P 500: Pre-Crisis Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.00043	0.000283	1.519765	0.129
R_{t-1}	-0.045813	0.048543	-0.943752	0.345
		Variance Equation		
ω	2.87E-06	2.37E-06	1.21423	0.225
α_{t-1}	0.035596	0.022218	1.602114	0.109
β_{t-1}	0.896381	0.07607	11.78365	0.000

MSCI ACWI: Pre-Crisis Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.000532	0.000219	2.431479	0.015
R_{t-1}	0.189755	0.044788	4.23671	0.000
		Variance Equation		
ω	1.34E-06	7.37E-07	1.816834	0.069
α_{t-1}	0.052924	0.020218	2.617658	0.009
β_{t-1}	0.904707	0.03849	23.50522	0.000

S&P 500: Crisis Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	3.57E-05	0.000566	0.063039	0.949
R_{t-1}	-0.169797	0.051731	-3.28229	0.001
Variance Equation				
ω	1.77E-06	1.28E-06	1.390438	0.164
α_{t-1}	0.100466	0.021213	4.73601	0.000
β_{t-1}	0.902434	0.021065	42.83978	0.000

MSCI ACWI: Crisis Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	4.90E-05	0.000428	0.11446	0.909
R_{t-1}	0.135564	0.045171	3.001112	0.003
Variance Equation				
ω	1.38E-06	1.01E-06	1.37298	0.169
α_{t-1}	0.137248	0.027172	5.051159	0.000
β_{t-1}	0.868213	0.024378	35.61466	0.000

S&P 500: Post Crisis Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.000887	0.000218	4.06913	0.000
R_{t-1}	-0.033987	0.030538	-1.112916	0.266
			Variance Equation	
ω	3.02E-06	6.23E-07	4.848868	0.000
α_{t-1}	0.119091	0.014371	8.287085	0.000
β_{t-1}	0.851438	0.016729	50.89741	0.000

MSCI ACWI: Post-Crisis Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.000483	0.000193	2.499849	0.012
R_{t-1}	0.152393	0.029347	5.192876	0.000
		Variance Equation		
ω	7.91E-07	2.28E-07	3.465072	0.001
α_{t-1}	0.065808	0.007746	8.495996	0.000
β_{t-1}	0.923769	0.008534	108.2498	0.000

Domestic: Full Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.001077	0.00018	5.980357	0.000
R_{t-1}	-0.059945	0.022556	-2.657569	0.008
Variance Equation				
ω	2.53E-06	4.85E-07	5.21533	0.000
α_{t-1}	0.109033	0.010971	9.937853	0.000
β_{t-1}	0.874337	0.012192	71.71488	0.000

Foreign: Full Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.001066	0.000182	5.852881	0.000
R_{t-1}	-0.029871	0.022418	-1.332417	0.183
Variance Equation				
ω	2.03E-06	4.14E-07	4.909422	0.000
α_{t-1}	0.083485	0.007922	10.53788	0.000
β_{t-1}	0.90138	0.008689	103.7325	0.000

Domestic: Pre-Crisis Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.001174	0.000343	3.425449	0.005
R_{t-1}	-0.066939	0.047693	-1.403539	0.161
Variance Equation				
ω	3.07E-06	1.79E-06	1.710267	0.087
α_{t-1}	0.059256	0.026907	2.20222	0.028
β_{t-1}	0.898364	0.045725	19.64698	0.000

Foreign: Pre-Crisis Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.001041	0.000345	3.013359	0.003
R_{t-1}	0.010564	0.047977	0.220195	0.826
Variance Equation				
ω	2.02E-06	1.33E-06	1.511585	0.131
α_{t-1}	0.026502	0.014895	1.77928	0.075
β_{t-1}	0.942622	0.031192	30.22005	0.000

Domestic: Crisis Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.000608	0.000612	0.993979	0.320
R_{t-1}	-0.157333	0.050632	-3.107378	0.002
Variance Equation				
ω	3.14E-06	2.10E-06	1.500144	0.134
α_{t-1}	0.114915	0.026356	4.360141	0.000
β_{t-1}	0.885259	0.027187	32.56205	0.000

Foreign: Crisis Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.000952	0.000567	1.679129	0.093
R_{t-1}	-0.126394	0.048131	-2.62605	0.009
Variance Equation				
ω	3.06E-06	1.87E-06	1.637714	0.102
α_{t-1}	0.11423	0.026891	4.247841	0.000
β_{t-1}	0.884708	0.027161	32.57296	0.000

Domestic: Post-Crisis Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.001082	0.00023	4.710968	0.000
R_{t-1}	-0.020649	0.029366	-0.70317	0.482
Variance Equation				
ω	3.43E-06	8.01E-07	4.289054	0.000
α_{t-1}	0.11717	0.015064	7.778109	0.000
β_{t-1}	0.852599	0.018117	47.06027	0.000

Foreign: Post-Crisis Period

Variable	Coefficient	Std. Error	z-Statistic	P
κ	0.001053	0.00024	4.378737	0.000
R_{t-1}	-0.004675	0.030347	-0.154051	0.878
Variance Equation				
ω	2.73E-06	6.61E-07	4.124923	0.000
α_{t-1}	0.09168	0.011461	7.999108	0.000
β_{t-1}	0.883255	0.012805	68.97807	0.000

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