Drew University
College of Liberal Arts

Portfolio Construction with Equity Linked Structured Products

A Thesis in Economics

by

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Abstract

The objective of this thesis is to analyze equity linked structured products with different forms of principal protection features. This exercise is worth undertaking given the recent developments in the financial markets. In fact, the ongoing crisis has raised concerns over both the relevance of some financial products and their ability to preserve investors' savings while yielding a return. In investing the construction of structured products, I first study the payoffs; valuation and market risks associated with the equity linked structured products on a stand-alone investment basis. From this first analysis the conclusion is that on a stand- alone basis equity linked structured products with principal protection are very attractive investment strategy – protecting the principal invested while yielding returns on the upside. Next, I investigate how equity linked structured products affect the risk/return profile of a traditional 60 per cent equity and 40 per cent bond portfolio by using simulation and scenario analysis. From the results I conclude that equity linked structured products with principal protection do not necessarily improve the portfolio performance and efficiency of a traditional portfolio because of the opportunity costs associated with them.

CONTENTS

1 Introduction

1.1 Background

Equity linked structured products have grown tremendously in the US retail market in the past decade. The goal of this study is to understand how Wall Street investment banks construct these products, what underlying risks they pose, and how these products fit into an investor's portfolio with traditional assets such as stocks and bonds. On a stand alone basis, the structured products under investigation in this paper offer a means of protection for part of or the entire capital originally invested. In the context of capital protected structured product, an investor may have as much as a hundred percent of his invested capital protected, which is the reason why a lot of investors prefer structured product investments and consider them worth the investment purely because of the possibility of a high profit with very little risk involved. One would expect that because of the principal protection and the high upside participation, the integration of principal protected structured products in a traditional portfolio would enhance the risk/return profile of the portfolio. This study analyzes the risk/return profile of structured products as a part of a traditional portfolio; So far, no such study has been conducted. Through Monte Carlo simulation I find that in a traditional portfolio of 40% bonds and 60% stocks, the integration of principal protected structured products does not necessary enhance the risk/return profile of the portfolio as it would be intuitively expected the structured products analysis on a stand alone basis.

1.2 Purpose

In this thesis, I illustrate the construction of equity- linked structured products and analyze the underlying risk factors associated with these products. In addition, I study the benefits of incorporating structured products in a traditional stock/bond portfolio by utilizing Monte-Carlo simulation in Excel VBA. Specifically, I compare the payoff distribution of portfolios with structured products and portfolios without structured products and study the risk/return impact of structured products on a traditional 40% bonds and 60% stocks portfolio.

The focus in this work is primarily on the equity linked (with stock or stock indexes as an underlying) growth products with different types of protections - fully principal protected note (PPN) and partially protected or buffered note (BN). Equity linked principal protected notes are among the most popular forms of structured products in the US retail market. For this paper I decided to study the (PPN) and the (BN) because of their attractiveness to retail investors in current risk averse market conditions.Their popularity in the US retail market is based on the fact that they offer full or partial capital protection while promising handsome return on the upside. Principal protected structured products are a combination of derivatives[1] and underlying financial instruments[2]. In particular, PPN's and BN's can be interpreted as a combination of a zero coupon bond and call/put options on the S&P500 Index.They have special risk/return profiles that might not be attainable by investing directly in the capital market, as direct investment also adds significantly to the transaction cost.

1.3 Literature Review

This section details prior research that was carried out in the area of structured products within the European and US markets. Little emphasis was

[1]See Appendix A9

[2]Underlying assets could be equity, commodities, interest rates, corporate credits or foreign exchange.

paid to structured products in the literature although they emerged in the US market in 1987. There have been only a handful of investigations in the US concerning valuation of structured products, all of which go back to a time when these products were first launched (Wohlwend 2003).

Structured products are not derivatives, but certainly stemmed out of the massive financial engineering[3] the industry experienced since the pricing of options was introduced by Black and Scholes in the 1973 [2]. Derivatives have become an important and controversial part of the financial system. The vast amont of literature written about derivatives reveal common concerns in regard to the size, complexity and risk associated with these securities. As we have experienced in the past two years, all these factors associated with derivatives have substantially increased the risk of collapse of the world financial system. The studies conducted directly on structured products as a separate asset class are not many. Most of the investigations deal specifically with valuations of different classes of structured products or structured products valuations in a specific market. Other studies look at cognitive factors affecting the valuation of structured products. Chen and Kensinger (1990), analyze structured products with principal protection issued by commercial

[3]See Appendix A6

banks at the time of the product issuance. Chen and Sears (1990) investigate the pricing of one structured product with principal protection in both the primary and the secondary market. Chen and Chen (1995) look into the secondary market valuation of one structured product without principal protection. For the German market, Wilkens (2003) examines the pricing of 900 structured products during a time frame of 22 days. Another study for the German market by Stoimenov and Wilkens (2005), analyzes the pricing of 2566 structured products on a single day. There are two studies conducted for the Swiss market. Wasserfallen and Schenk (1996) investigate the price behavior of thirteen structured products with principal protection. Burth (2001) examines pricing at issuance for 275 products with no principal protection. Gruenbichler and Wohlwend (2005) look at the pricing of 192 structured products with no principal protection and survey the valuation of the products in both the primary and the secondary market in Switzerland. In all of the above mentioned studies it was found that overall structured products were overvalued at the time of issuance.

In more recent studies, Breuer and Perst (2007) look at the market success of discount reverse convertibles (DRC's) and reverse convertible bonds

(RCB's)[4]. In this study it is found that in theory the market success of DRC's would be overestimated and the success of RCB's would be underestimated in comparison to a situation with bounded rationality[5]. A similar study was done in 2008 where Hens and Rieger (2008) develop a theoretical framework for the design of an optimal structured product and analyze the maximum utility that could be gained. They demonstrate that even the most successful structured products are not optimal for a perfectly rational investor. Their attractiveness and success comes out of the fact that investors are likely to perform probability miscalculation and are loss-averse.

Wallmeier and Diethelm (2008) investigate the most successful Swiss structured products - reverse convertible on multiple assets with conditional capital protection (MBRC's). They provide an empirical study of the valuation of this class of structured products and obtain an average overpricing of at least 3.4%. They also find that the overpricing is positively related to the coupon level of the product. This result indicates that investors tend to put

[4]DRC's and RCB's are examples of structured products combining a zero coupon bond plus a short position in put options on stock. (Breuer and Perst)

[5]The concept of "bounded rationality" revises the assumption that humans are rational decision makers to account for the fact that perfectly rational decisions are often not feasible in practice due to the finite computational resources available for making them.

much value on the sure coupon and underestimate the risk involved. This conclusion is then instrumental in explaining the success of this product class in Switzerland.

Eberlein and Madan (2009) investigate different structured products pricing techniques and models. In their study they look at the Sato processes to value equity linked structured products, and find that different products should be priced using different models.

The main difference between the study presented in this paper and earlier studies lies not only in illustrating the construction of structured products and the relevant analysis of market risk associated with them, but also in the investigation of the benefit and the possible shortfall of structured products as a part of an investor's portfolio. All previous studies of structured products concentrate mainly on their valuation on a stand-alone basis and do not analyze the performance of structured products as an integral part of an investors portfolio. The goal of this work is to compare the payoff distribution of portfolios with structured products and portfolios without structured products. While it is generally perceived that structured products with principal protection outperform other traditional investments strategies, my Monte Carlo simulations show that this is not the case if the benefits of structured products are evaluated as part of a portfolio.

1.4 What is a Structured Product?

Stoimenov and Wilkens (2005), define structured products to be a combining at least two single instruments of which at least one is a derivative [23]. The term "structured product" is the name given to an investment product that provides a return that is pre-determined with reference to the performance of one or more underlying markets. The performance of a structured product is therefore based only on the performance of this underlying and not on the discretion of the product provider. Often, but not always, the product relies on the use of derivatives to generate the return. Structured products are therefore financial instruments that are designed to change the risk and return profiles of traditional financial assets in order to suit the needs of investors with different risk appetites. To change the risk and return profiles of traditional financial assets, structured products are generally constructed by combining traditional financial assets,(such as stocks and bonds) with financial derivatives, (such as options and swaps).[6] As a result, unlike mutual funds, structured products do not expose the investor to management skills of

[6] See Appendix A1-A3

the investment manager - these products work the way they were structured prior to putting them in action. The performance of a structured product is therefore based only on the performance of the underlying and not on the discretion of the product provider.

Structured products have experienced a massive growth in Europe in recent years not only in quantity but in variety. Specifically, the Swiss market for structured products is characterized by wide products diversity. Categorizing is not easy. Wohlwend and Gruenbichler (2005)categorize structured products of two kinds.The first category is structured products with capital protection and the second category is structured products with no capital protection. Products with no capital guarantee can be further subdivided into products without coupon payments, products with coupon payments and products with exotic characteristics[7] [11].

Often structured products are mistaken to be derivatives[8] or "retail derivatives". Structured products have derivative(s) as a main component, but as mentioned earlier this is not always the case; In addition, structured products also combine traditional financial assets like stocks and bonds.

[7] Exotic as opposed to vanilla refers to the fact that the payoff is not standard, as is the case for a regular call option. See Appendix A3.

[8] See Appendix A9

In general, structured products can be linked to any kinds of asset classes with different types of payoff depending on the risk appetite of investors. Therefore, in terms of asset classes, structured products can be classified into equity linked, interest rate linked, credit linked, FX linked, commodity linked or even hedge fund linked. In terms of payoffs, structured products can be divided into two main categories: growth (which may provide an element of protection) and income (which provides a fixed high income but with risk to the capital return) [23],[25].

1.5 US Structured Products Market Overview

It is worth mentioning that the global structured products market maybe big in absolute terms, but relatively small in comparison to the global derivatives market. The derivatives market had grown fivefold from 2002 to 2008 - $106 trillion to $513 trillion respectively [16]. Compared to the derivatives market the structured products global market is $313bn[9] with the Swiss market ranking among the largest in the world [23],[26].

The total size of the US structured products market is $75bn as of year 2005. The US market lags behind Europe in terms of sales, which had more

[9]Estimates for year 2005

than double the sales of the US in 2005 at $169bn. Probably the main reason for the slower development of the industry in the US compared Europe is that US investors have traditionally invested in equity via stocks and mutual funds. They therefore prefer direct investment compared to partial exposure via structured products.US has been growing strongly in recent years, and given the size of its economy as a whole has the potential to become a major international player. According to Structured Retail Products structured products is the fastest growing investment class in the US. Between 2000 and 2002 most US products offered full principal protection but since then there has been a swing to non-protected products, primarily reverse convertibles[10]. Most products tend to have maturities of less than five years, the exception being index-linked annuities with surrender periods of between five to over ten years. Data suggests that since 1996 till 2006 there is steady growth in issues in Belgium, The Netherlands and the UK, a stabilization in France and rapid growth in the German, the Italian and the US market. In terms

[10]Reverse convertible security or reverse convertible is a short-term note linked to an underlying stock. The security offers steady stream of income due to the payment of a high coupon rate. At maturity, the investor will receive either 100% of the par value or a predetermined number of shares of the underlying stock, in addition to the stated coupon payment.[20]

of volumes, there is a steady rise in most markets and rapid growth in the German and US markets [10],[23].

In 2008 through early November, nearly $34 billion worth of structured products were sold to small U.S. investors, surpassing the $33.5 billion sold in all of 2007. All products that are to be sold to retail investors must be registered with the SEC, as non-registered products in note form can be sold only to accredited investors such as high net worth individuals or institutional investors.[20]

A liquid secondary market is very important to the US investor and this is one of the reasons why listing a product is popular. Even issuers of unlisted products often offer some form of secondary market. Market-linked CDs however are generally simple principal-protected deposits usually linked to an equity index (normally the S&P500 or DJIA) for which no secondary market is available. These CD products are sold by large national banks as well as local Savings Banks and benefit from FDIC insurance [7],[23].

In 2009 the US structured product market will be facing many challenges. Among the most important is regaining investor confidence regarding issuer credibility. With the default of Lehman Brothers, investors have been left feeling uneasy about debt-based capital protection. Other major hurdles in

2009, as noted by the structured product community, will be creating stronger ties with industry regulators, standardizing structured products disclosure statements, greater educational advertising, and a central directory listing of structured products providers, services and key professionals [23].

2 ANALYSIS: A Focus on Structured Products With Principal Protection

In this section, I am going to analyze two types of equity linked products with principal protection in detail, namely the principal protected note (PPN) and the buffered note (BN). Each note is linked to the S&P 500 Index with a maturity of five years. In the following, I am going to illustrate the payoff of each note and explain in details how each note is constructed and priced at inception. In addition, I will also discuss the market risk factors associated with each note and how these risk factors will affect the valuation of each note prior to maturity. The main difference between the two principal protected equity linked notes is that the PPN is 100% principal protected note and the BN has 20% protection on the principal invested.

2.1 Principal Protected Note (PPN)

2.1.1 Payoff

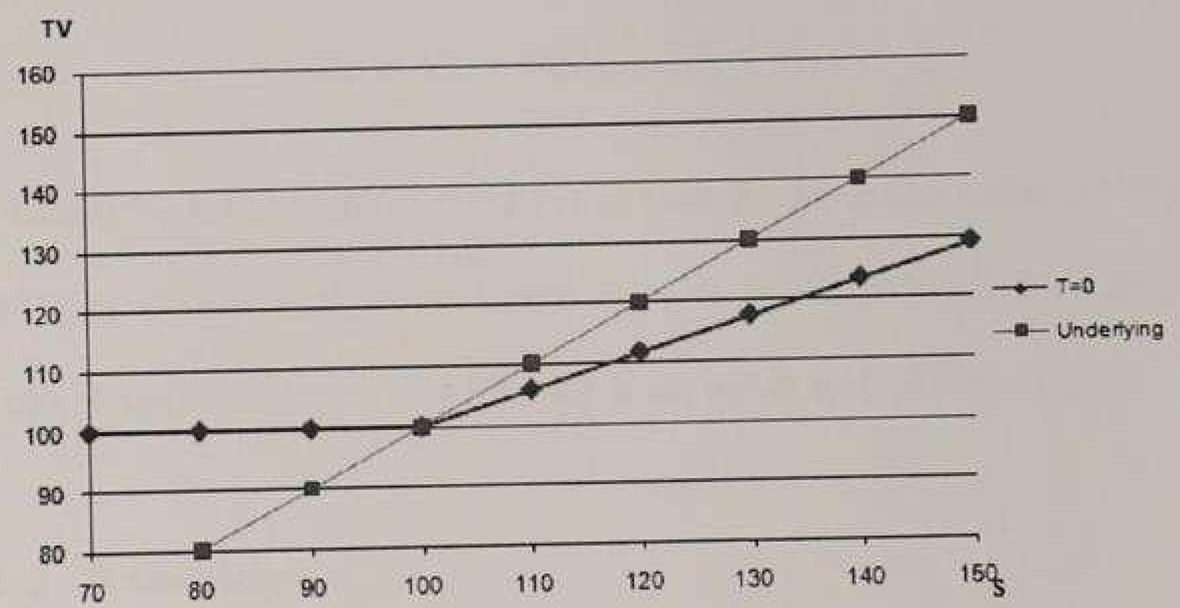

Figure 1

The figure above shows the payoff of a 5-year PPN linked to the S&P500 Index at maturity versus the index itself. The PPN is 100% principal protected with a participation rate of 92%, which means that at maturity, if the S&P500 Index falls below the initial level (i.e. 100), PPN investors will receive the principal back. If the S&P500 Index rises above the initial level, for every 1% increase of the index, the PPN will increase by 0.92%.

Therefore, the diagram shows that the PPN will outperform the S&P500

Index if the index falls 5 years from now (i.e. the value added as principal is protected), but it will under perform if the index rises over the next 5 years.

At maturity, the value of PPN is calculated as:

$$V_T = 100 + 0.92 \max(S_T - S_0, 0) \tag{1}$$

Where V_T is the value of the PPN at time T, S_T is the value of the S&P 500 Index at time T, S_0 is the initial value of the S&P 500 Index and is set to 100, T is the maturity of PPN and is set equal to 5 (five years).

2.1.2 Valuation

How do issuers determine the participation rate, in this case 0.92, of a PPN?

At inception, issuers generally issue structured notes at par less underwriting fees, just like any company issues common stocks or bonds in the primary market. In the case of PPN, PPN is issued at par less underwriting fees under the market conditions at inception. In order to calculate the participation rate, we need to understand the building block of a PPN.

A 5-year PPN linked to the S&P500 Index is composed of a 5-year zero coupon bond plus an ATM (At The Money) call option linked to the S&P500

Index[11]. Assuming a PPN is priced at par (i.e. $100) less underwriting fee of 3.5%, issuers will split the proceeds into a zero coupon bond and a certain number of call option, and we need to calculate how much is the zero coupon bond and the call option worth in the market.[12]

In mathematical form, the pricing equation is:

$$V_0 = L * (1 - fee) = \eta * L * e^{-rT} + \lambda * CR * C_{k_1} \tag{2}$$

Where $V_0$is the initial value of a PPN, η is the protection level and it is equal to 100%, r is the spot rate for the corresponding time to maturity, L is the notional and is equal to 100, CR is equal to L/S_0, $S_0$is the initial S&P500 Index level, and is set to 100, k_1 is the strike level and is equal to S_0, and $C_{k_1}$is the value of the call option struck at k_1.

I use Black - Scholes formula[13] to value the call option, and solve for the participation rate (λ) using the market parameters below [14]:

[11]Please see Appendix A4 for details.

[12]Average underwriting fee of a 5-year PPN is 3.5% among investment banks in the US.

[13]See Apendix A6

[14]For this thesis, I assume financial markets are efficient and are at long term equilibrium, and I use the past 20 years average as the proxy for each market parameter. In reality, issuers use the current market parameters to determine the participation rate.

5 year interest rate = 5.9%

5 year implied volatility = 20%

S&P500 annual dividend yield = 2%

(Source: Bloomberg)

After inserting the parameters into the above equation, we obtain the following:

$$\text{zero coupon bond} = Le^{(-rT)} = 74.45 \quad (3)$$

$$\text{ATM call option} = C(k_1) = 23.94 \quad (4)$$

Solving for λ, we get the participation rate equal to 92.07%

Therefore, upon receiving the proceeds, issuers will invest the proceeds into a zero coupon bond and 0.92 ATM call option (i.e. 1 note = 1 bond + 0.92 call option and 100 notes = 100 bonds + 92 call options). At maturity, the zero coupon bond will be worth par and provide full protection

Since the markets are volatile, especially during the current credit crisis, the participation rate tends to change every day and can be significantly different from the long term equilibrium value.

for the principal invested. If the price performance of the S&P500 Index is positive, then the call option will be exercised and return the performance of the S&P500 Index stock to investors. However, since only 0.92 call option is invested, investors will only get 92% exposure to the S&P500 Index at maturity. Therefore, PPN can always return the principal invested at maturity regardless of the performance of the underlying asset, while capturing the upside performance of the underlying.

2.1.3 Risk Analysis

There are five market risk factors in PPN, which are price performance of the underlying, time remaining to maturity, volatility, interest rates and dividend yield. To analyze each risk factor, we vary the level of each risk factor in the equation below and recalculate the theoretical value (TV) of the PPN. Here I calculate the theoretical value because this value is based on quantifiable risks within the model. In reality, the value of the notes depends also on the liquidity provided by the issuer and the market conditions.

In order to analyze the market risks of PPN, we need to value PPN prior to maturity and the formula is as follows:

$$V_t = \eta * L * e^{-r(T-t)} + CR * \lambda * C_{k_1} \tag{5}$$

Underlying and time to maturity

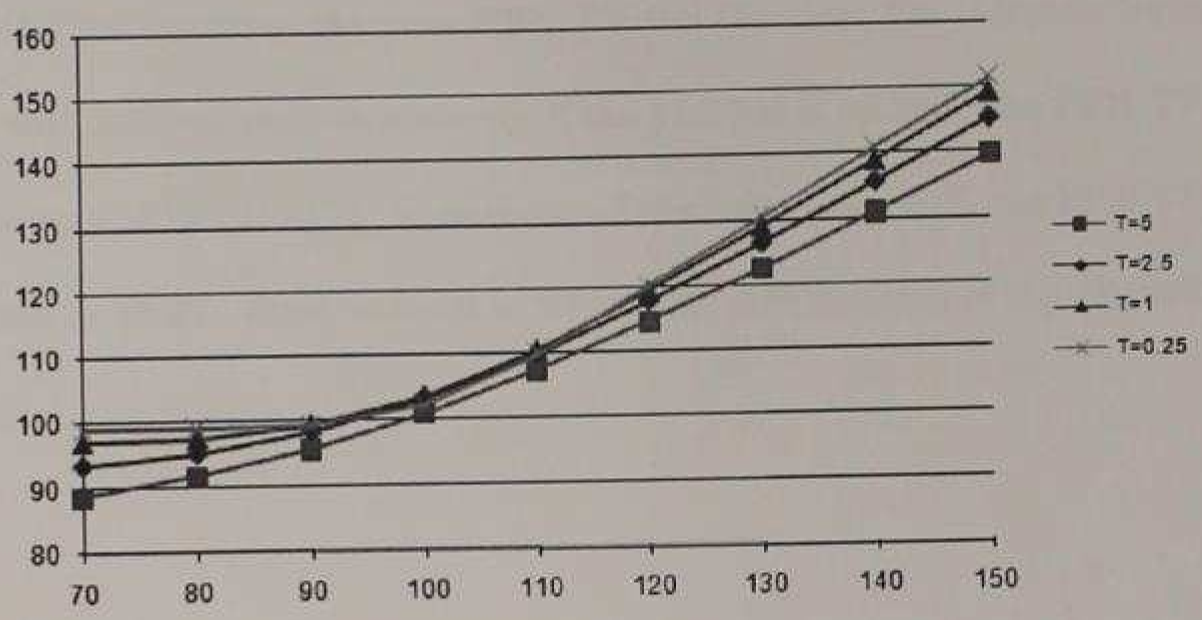

Figure 2

The figure shows the underlying sensitivity of PPN prior to maturity. Prior to maturity, if the underlying is below the initial level 100, the TV of PPN is below par, which means that PPN is not principal protected before maturity! For example, let's look at a PPN with 2.5 years prior to maturity (i.e. T=2.5). If the S&P500 Index is down 10% (i.e. at 90), the PPN TV will be down 2% (i.e.98). The closer to maturity, the more protection PPN will offer.

For example, for PPN with 0.25 year left to maturity, if the S&P500 Index is down 10%, the PPN will be down only 1% (i.e. 99). At maturity, the PPN will be traded at 100, which provides the principal protection.

On the upside, if the underlying is up, PPN TV will also increase. The closer to the maturity, the more PPN TV will increase. For instance, for a PPN with 2.5 year prior to maturity, if the S&P500 is up 20%, the PPN TV will be up 17.75% (117.75). At maturity, if the index is up 20%, the PPN TV increase by 18.4%, which is equal to 92% of the 20% increase of the S&P500 Index.

Volatility

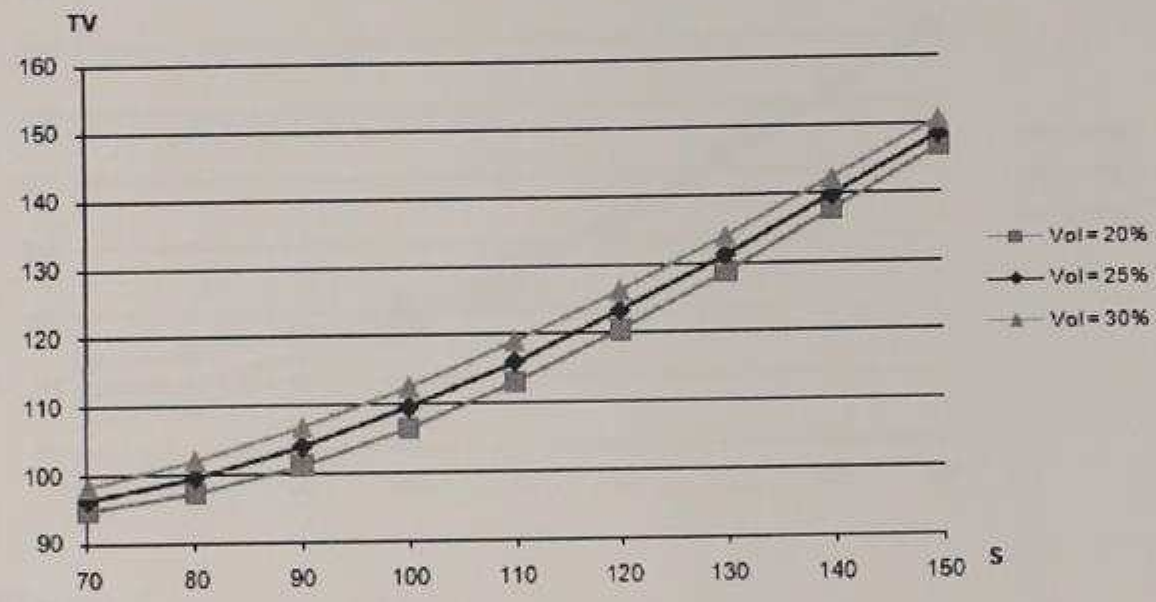

Figure 3

The figure above shows the volatility[15] sensitivity of PPN prior to maturity. The higher the volatility, the higher the value of PPN because the higher the volatility, the higher the value of the embedded option[16]. For instance, at the initial index level (100), with volatility at 20%, 25% and 30%, the TV of the note increases to 105,107 and 110 respectively.

Interest rate

[15]Volatility used in this paper is the average 5 year historical volatility of the S&P500 Index using past 20 years of data.

[16]See Appendix A5

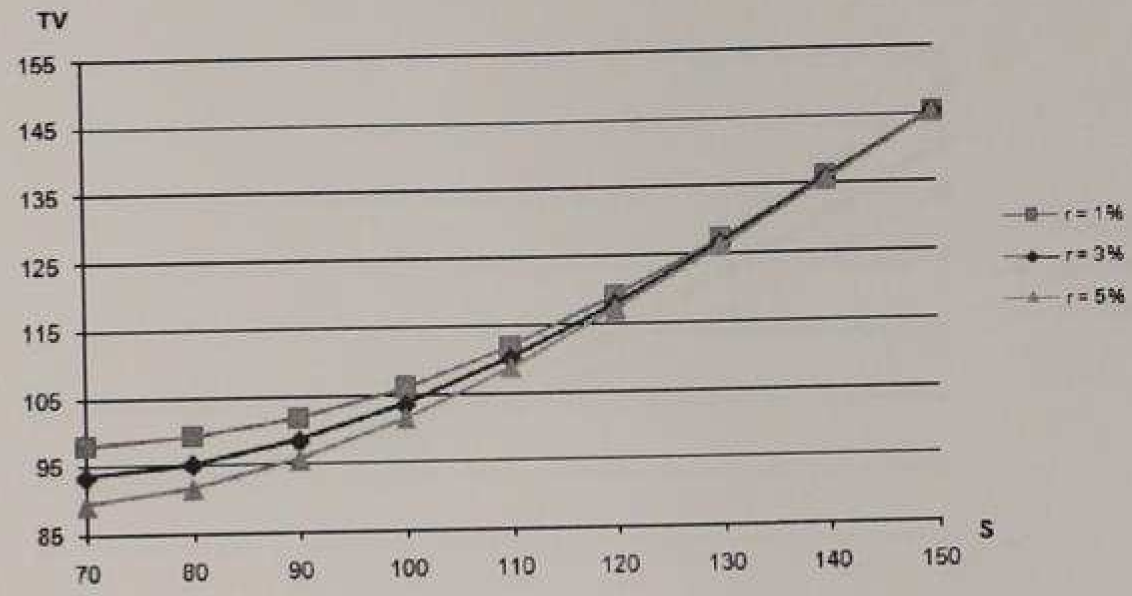

Figure 4

Fig 4 shows the interest rate[17] sensitivity of PPN prior to maturity. The lower the interest rate, the higher the value of PPN because the lower the interest rate, the higher the value of the zero coupon bond[18] . For instance, at the initial index level (100), if the interest rate decreases from 5% to 1%, the PPN TV will increase from 100 to 105.

Dividend Yield

[17]The discount rate, r, used in the paper is the swap rate, not the Treasury rate. Swap rate refers to the interbank lending rate with AA credit.

[18]The zero-coupon 5-year bonds is a zero coupon bond backed by the issuer, therefore, it involves credit risk,

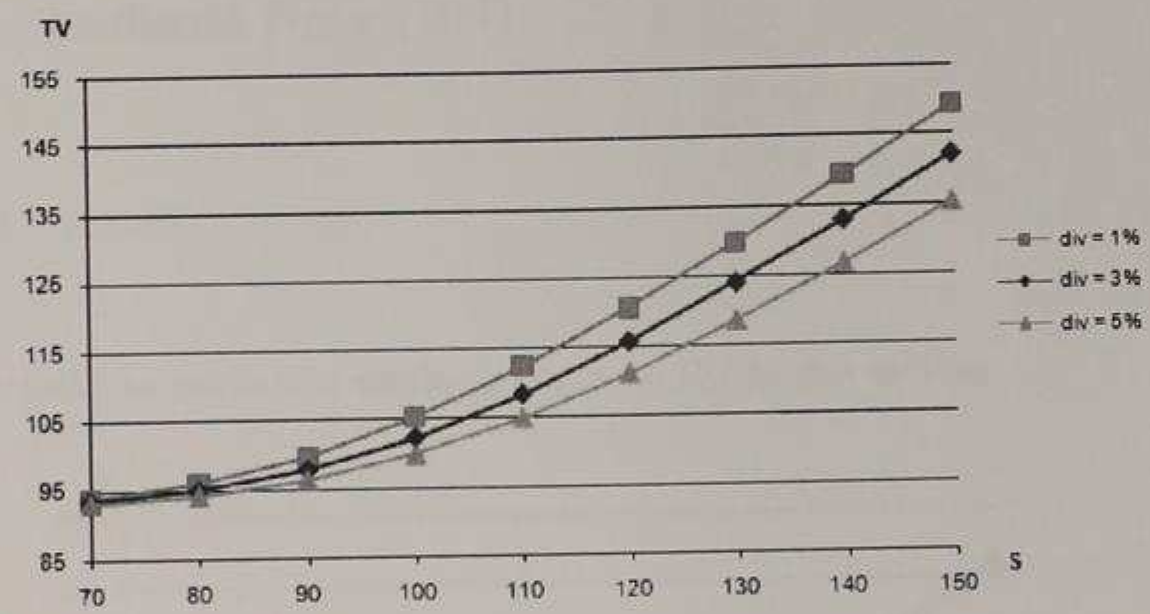

Figure 5

The figure above shows the dividend yield sensitivity of PPN prior to maturity. The higher the dividend yield, the lower the value of PPN because the higher the dividend yield, the lower the value of the call option[19]. For instance, at the initial index level (100), if the dividend yield decreases from 1% to 5%, the PPN TV will decrease from 101 to 95.

[19]See Appendix

2.2 Buffered Note (BN)

2.2.1 Payoff

I am going to perform a similar analysis on BN in this section.

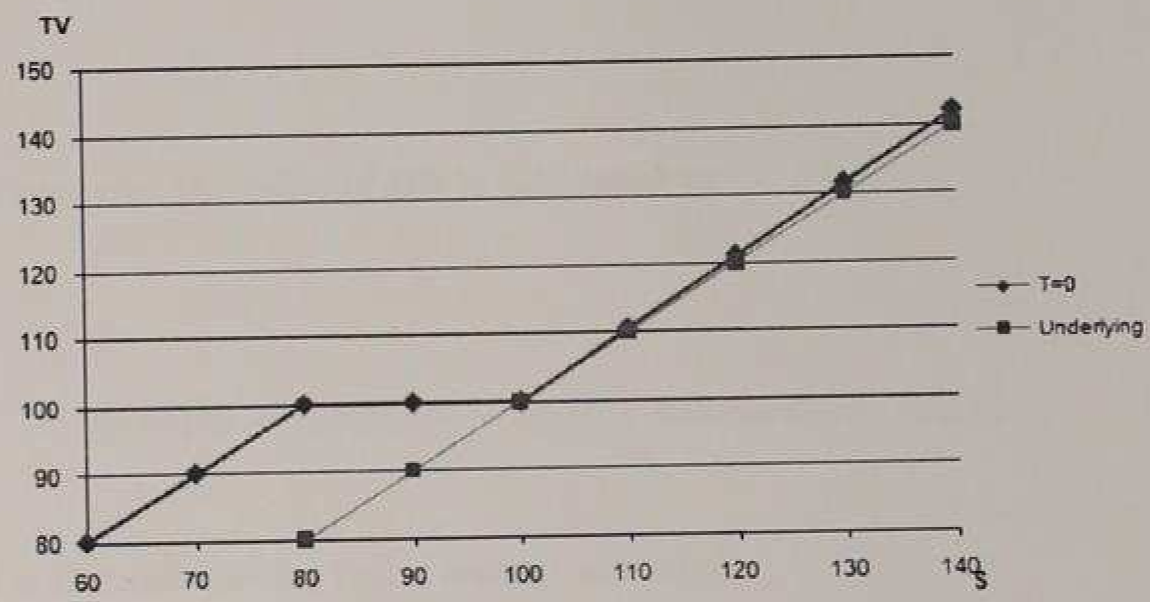

Figure 6

The figure above shows the payoff of a 5-year BN linked to the S&P500 Index at maturity versus the index itself. The BN is 20% principal protected with a participation rate of 105%, which means that at maturity, if the S&P500 Index falls below the initial level by less than 20% investors are principal protected. If the index falls more than 20%, investors will lose part of the principal. On the other hand if the S&P500 Index rises above the initial level, for every 1% increase of the index, the BN will increase by 1.05%.

Therefore, if the index is down, BN investors will have higher return than the index investors. If the index is up, BN investors will also outperform index investors (excluding dividends) because of higher than 100% participation rate.

At maturity, the value of BN is calculated as:

$$V_T = 100 - \max(0.8S_0 - S_T, 0) + 1.05 \max(S_T - S_0, 0) \tag{6}$$

S_T is the value of the index level at maturity.

2.2.2 Valuation

How do issuers determine the participation rate (i.e. 1.05) of BN?

A 5-year BN linked to the S&P500 Index is composed of a 5-year zero coupon bond minus one 20% OTM put option plus a certain number of ATM call options linked to the S&P500 Index[20]. Assuming BN is priced at par (i.e.

[20] Please see Appendix A4 for details.

$100) less underwriting fee of 3.5%, issuers will structured split the proceeds into a zero coupon bond, an OTM (Out of The Money) put option and a certain number of ATM call options. To calculate the number of ATM call options I need to calculate how much the zero coupon bond, the call option and the put option are worth in the market.[21]

In mathematical form, the pricing equation is:

$$V_0 = L(1 - fee) = L * e^{-rT} + CR * (-P(K_1) + \lambda * C(K_2)) \qquad (7)$$

Where r is the structured product rate for the corresponding maturity, $P(K_1)$ is the value of the put option struck at K_1, $C(K_2)$ is the value of call option struck at K_2. K_1 is the strike of the put option and is equal to $(1 - 0.2)S_0$, K_2 is the strike of the call option and is equal to S_0.

I use Black - Scholes formula to value the put and the call option, and solve for the participation rate (λ) using the market parameters below:

5 year interest rate = 5.9%

5 year implied volatility = 20%

S&P500 annual dividend yield = 2%

[21] Average underwriting fee of a 5-year PPN is 3.5% among investment banks in the US.

(Source: Bloomberg)

After inserting the parameters into the above equation, I obtain the following:

$$\text{zero coupon bond} = Le^{(-rT)} = 74.45 \tag{8}$$

$$\text{ATM call option} = C(k_1) = 23.94 \tag{9}$$

$$\text{ATM put option } = C(k_2) = 3.04 \tag{10}$$

Solving for λ, we get the participation rate equal to 104.78%

2.2.3 Risk Analysis

Similar to PPN, there are five market risk factors in BN, which are price performance of the underlying, time remaining to maturity, volatility, interest rates and dividend yield. To analyze each risk factor, we vary the level of each risk factor in the equation below and recalculate the theoretical value (TV) of the BN.

In order to analyze the market risks of BN, we need to value BN prior to maturity and the formula is as follows:

$$V_t = L * e^{-r(T-t)} + CR * (P(K_1) + \lambda * C(K_2)) \tag{11}$$

Underlying and time to maturity

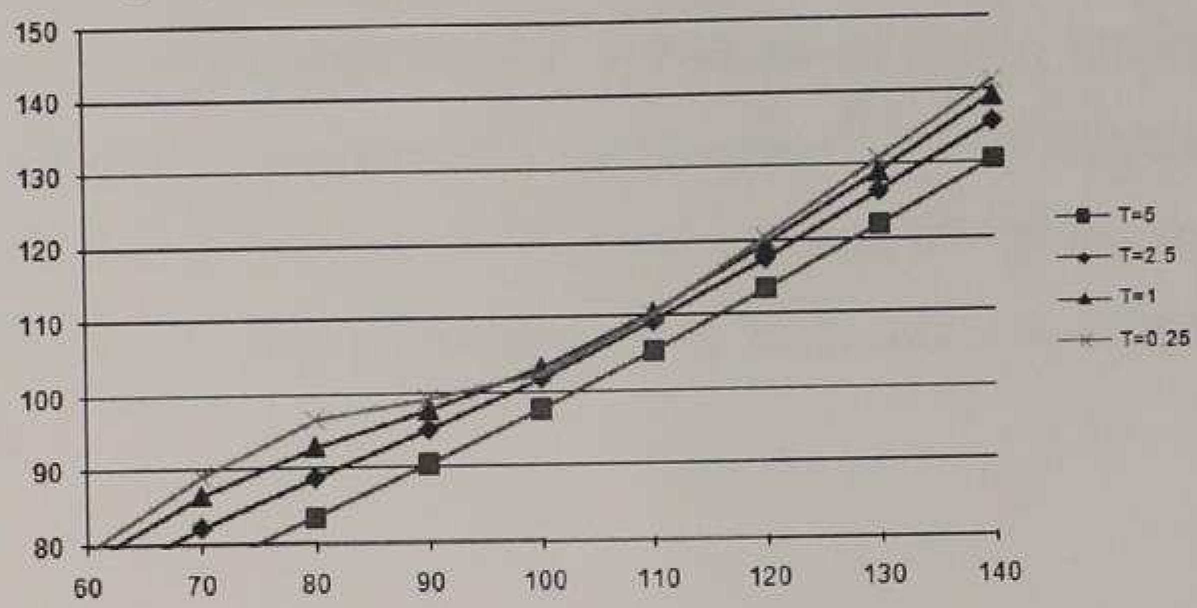

Figure 8

The figure above shows the underlying sensitivity of BN prior to maturity. Prior to maturity, if the underlying is below the initial level, the TV of BN is below par, which means that BN is not having partial principal protected before maturity. For example, let's look at a BN with 2.5 years prior to maturity (i.e. T=2.5). If the S&P500 Index is down 10% (i.e. at 90), the BN TV will be down 5% (i.e.95). The closer to maturity, the more partial protection BN will offer.

For example, for BN with 0.25 year left to maturity, if the S&P500 Index is down 10%, the BN will be down only 1% (i.e. 99). At maturity, the BN will be traded at 100, which provides 20% protection.

On the upside, if the underlying is up, BN TV will also increase. The closer to the maturity, the more BN TV will increase. For instance, for a BN with 2.5 year prior to maturity, if the S&P500 is up 20%, the BN TV will be up 18% (i.e.118). At maturity, if the index is up 20%, the PPN TV increase by 21% (i.e. 121), which is equal to 105% of the 20% increase of the S&P500 Index.

Volatility

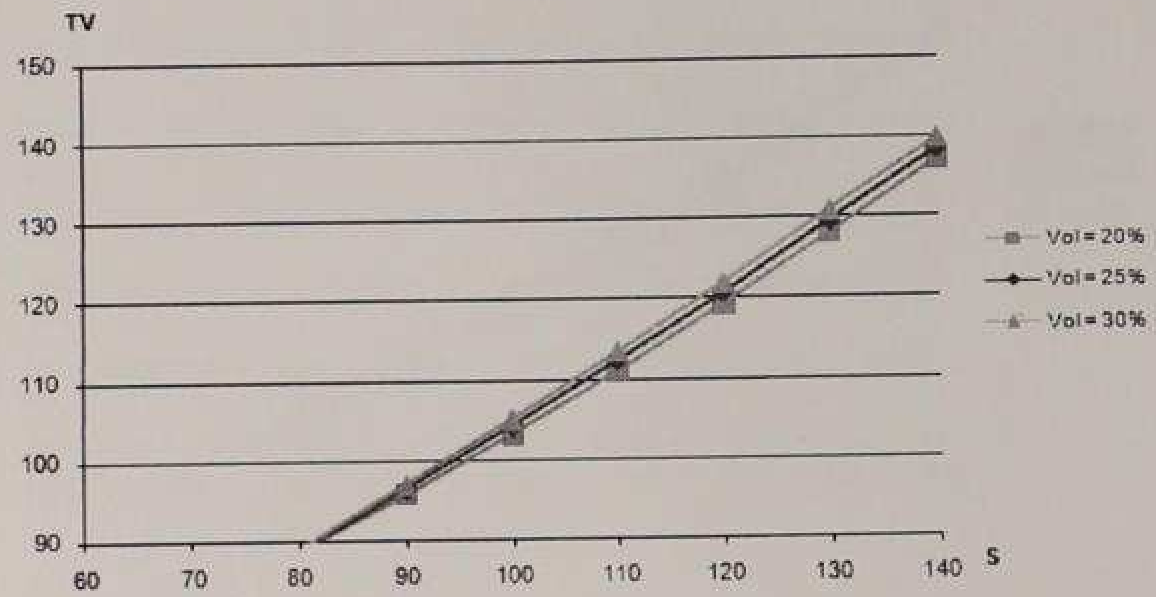

Figure 9

The figure above shows the volatility sensitivity of BN prior to maturity. There is small or no change in the TV for volatility change. For instance, at the initial index level (100) (time is fixed at 2.5), for volatility increase of 20%, 25% and 30% the value of the note increases from 103 to 104 to 105 respectively, which barely moves.

Interest rate

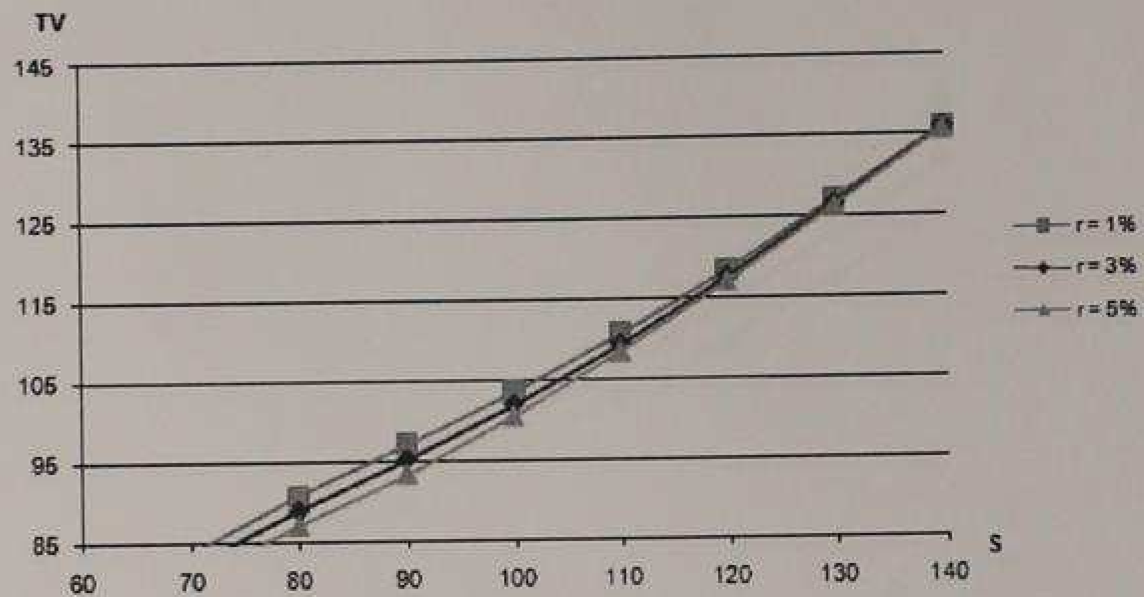

Figure 10

Interest rate has more impact on the downside than on the upside. For instance, at the initial index level (80), if the interest rate decreases from 5% to 1%, the BN TV will increase from 87 to 90 [22]. At index level of 120 TV stays around 120 if r decreases from 5% to 1%. Below 100 the bond part of the structured products dominates over the other components. Above the 100 level the call option becomes more valuable which counterbalances the effect of the bond and the put becomes worthless.

Dividend Yield

[22]The interest rate and the value of the note move in opposite directions due to the fact that the largest component of the note is the zero coupon bond = Le^(-rT).

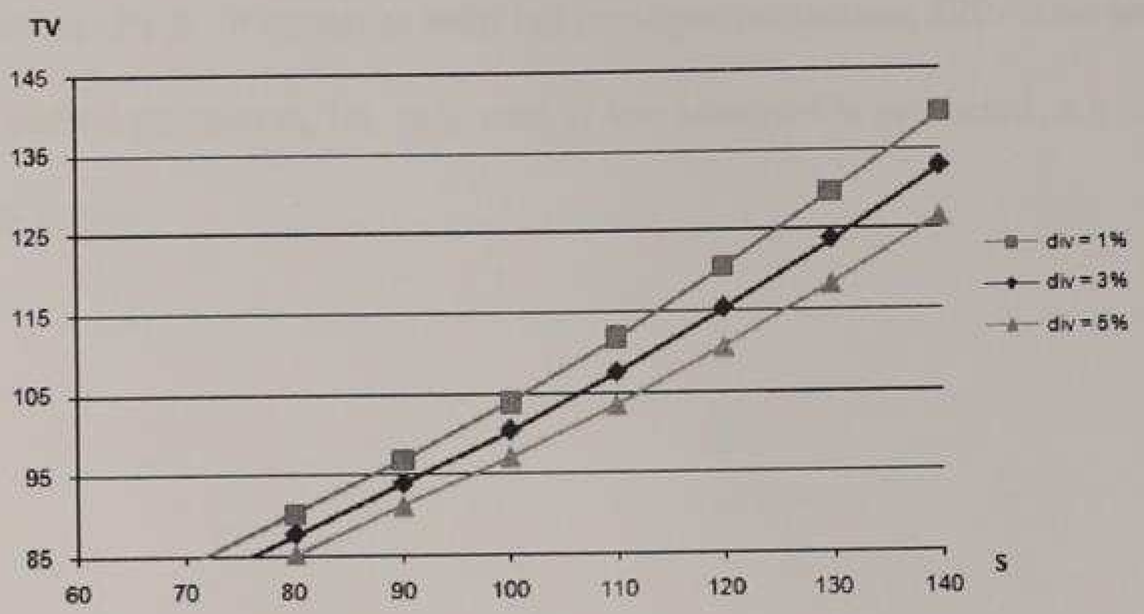

Figure 11

The figure above shows the dividend yield sensitivity of BN prior to maturity. The higher the dividend yield, the lower the value of BN because the higher the dividend yield, the lower the value of the call options and the higher the value of the put option[23]. For instance, at the initial index level (100), if the dividend yield decreases from 1% to 5%, the BN TV will decrease from 104 to 97. This is due to the fact that as dividend increases, the call options value decrease and the put option value increases. In this situation put option is sold and call option is long. BN decreases in value as dividend increases.

Based on the calculation above it is clear why investors would prefer one

[23]See Appendix A5

or another. PPN - if investors want full principal protection; BN - if investors want partial protection, i.e. only part of the principal is protected, e.g. 20% in this paper.

2.3 Liquidity Risk and Issuer Credit Risk

In addition to market risks, equity linked structured products investors also need to be aware of liquidity risk and issuer credit risk associated with these products. If investors want to redeem their structured products prior to maturity, they can sell back their notes to the issuers, and the issuers will buy back the notes under normal market conditions. In the case of very volatile markets, for example the week before Lehman Brothers filing bankruptcy in the Fall of 2008, issuers might not be able to fulfill their obligations to buy back their notes, and investors will be stuck with their structured products investments and will not be able to liquidate their positions when needed. In addition, investors need to be aware of issuer credit risk associated with structured products. In legal terms, structured products are senior unsecured debts issued by financial institutions, which means that if an issuer goes bankrupt, investors holding the structured product issued by that issuer may not get their principals back even though the products are principal protected. Therefore, investors face issuer default risk when they purchase structured products.

A perfect example is the Lehman Brothers bankruptcy in 2008, in which structured products investors worldwide faced significant amount of principal loss from structured products issued by Lehman Brothers. Immediately preceding the September '08 Lehman bankruptcy, investors could not sell their structured products linked to Lehman, there was no secondary market. Under the law the issuer is not obligated to provide liquidity. There are two different cases in which one could lose the principal invested in a 100% principal protected note. The first case - holding Lehman Credit linked notes. The second case - holding notes for which Lehman as the issuer. In the first case the notes could be sold to investors from banks all around the world, it is important to note that the bank selling the notes linked to Lehman's credit is not backing them, Lehaman is. This is how many investors lost a lot of money, they thought that the bank selling the notes is taking the risk. In fact, the bank just sold them, while the investors bear the risk associated with the notes. In second case, Lehman was not required to provide secondary market and there was none.

3 Portfolio Construction With Structured Products

After analyzing equity linked structured products on a stand alone basis, I would like to study the risk/return reward of combining equity linked structured products with a traditional asset portfolio to investigate if equity linked structured products with a traditional asset portfolio to investigate if equity linked structured products can be used to enhance investors' portfolio efficiency on a forward looking basis, assuming the markets are at equilibrium.

Specifically, I use Monte - Carlo simulation to analyze the risk/return profile of an equity/bond portfolio with each of the equity linked structured product mentioned in section 2 over an investment horizon of 5 years. For investors with moderate risk appetites, their asset allocation is usually 60% in diversified domestic equities and 40% in diversified investment grade domestic bonds as suggested by the majority of financial advisors in the industry, and such portfolio allocation is called "traditional portfolio". In this section, I am going to compare the "traditional portfolio" with an "Equity Linked Structured product (ELN) portfolio" which allocates 90% into traditional portfolio and 10% into each of the equity linked structured product linked to the S&P500 Index. In the simulation, I am going to simulate 3 portfolios:

1. Traditional portfolio: 60% equities/40% bonds

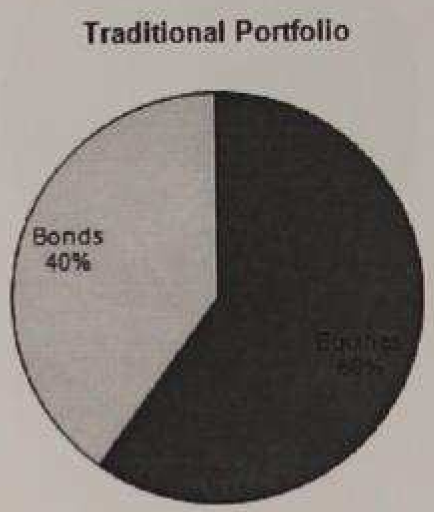

Figure 12

2. ELN portfolio I: 54% equities/36% bonds/ /10% PPN

ELN I Portfolio

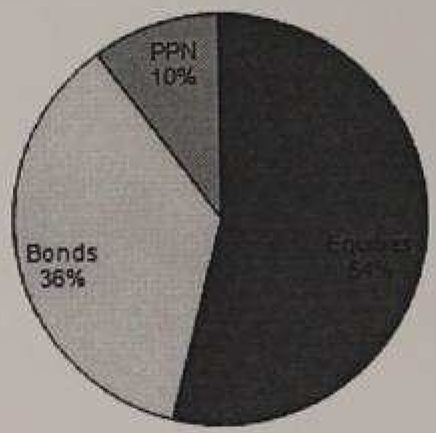

Figure 13

3. ELN portfolio II: 54% equities/36% bonds/10% BN

ELN II Portfolio

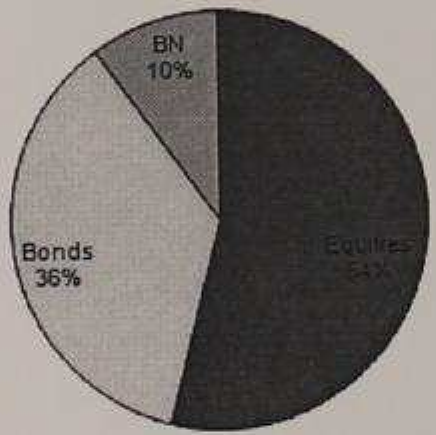

Figure 14

At the end of the simulation, I will be able to obtain the simulated return distributions of the three portfolios and compute the risk return statistics.

3.1 Monte Carlo Simulation

Monte - Carlo simulation is a numerical technique that estimates the future distribution of asset returns over a predefined investment horizon. This method involves generating thousands of assets paths (3,000 in this analysis) with monthly time steps that simulate all the possibilities that can happen in the future given the model assumptions. At the end of the simulation, I will be able to summarize the probability distribution of the asset returns, and calculate the expected returns and the risks of the portfolios. The input parameters for the simulation are the future expected returns and the volatilities of the assets, the correlation among the assets, and the asset returns dynamics over the investment horizon.

In this analysis, the assets are diversified domestic equities and diversified domestic bonds. As a proxy, I use the S&P500 Index to approximate the returns of the diversified domestic equities, and the BarCap US Agg Total Return Value Index to approximate the returns of the domestic investment grade bonds.

Assume the return dynamics for all assets are following geometric Brownian motion[24] with constant volatility and constant interest rates (i.e. Black -

[24]See Appendix

Scholes framework[25]). The asset dynamics can be described by the following stochastic differential equations:

$$dS/S = \mu_S dt + \sigma_S dW_S \tag{12}$$

$$dB/B = \mu_B dt + \sigma_B dW_B \tag{13}$$

$$E(dW_{S,} dW_B) = \rho dt \tag{14}$$

where

S = price of the S&P 500 Index

B = price of the BarCap US Agg Total Return Value Index

μ_S = expected price return of the S&P 500 Index

[25]See Appendix

μ_B = expected price return of the BarCap US Agg Total Return Value Index

σ_S = volatility of the S&P500 Index

σ_B = volatility of the BarCap US Agg Total Return Value Index

dW_S = Brownian motion of the S&P 500 Index with mean 0 and variance dt

dW_B = Brownian motion of the BarCap US Agg Total Return Value Index with mean 0 and variance dt

ρ = correlation between the retun of the BarCap US Agg Total Return Value Index and the S&P 500 Index

By Using Ito's Lemma on Equation 14, the equation becomes:

$$d\ln S = (\mu_S - \frac{1}{2}\sigma_S^2)dt + \sigma_S dW_S \quad (15)$$

Integrating both sides from 0 to T, we get

$$\ln S_T - \ln S_0 = (\mu_S - \frac{1}{2}\sigma_S^2)T + \int_0^T \sigma_S dW_S \quad (16)$$

$$S_T = S_0 \exp((\mu_S - \frac{1}{2}\sigma_S^2)T + \int_0^T \sigma_S dW_S) \tag{17}$$

Following the same procedure, equation 15 becomes:

$$d\ln B = (\mu_B - \frac{1}{2}\sigma_B^2)dt + \sigma_B dW_B \tag{18}$$

$$\ln B_T - \ln B_0 = (\mu_B - \frac{1}{2}\sigma_B^2)T + \int_0^T \sigma_B dW_B \tag{19}$$

$$B_T = B_0 \exp((\mu_B - \frac{1}{2}\sigma_B^2)T + \int_0^T \sigma_B dW_B) \tag{20}$$

Simulating the asset paths means discretizing equations 17 and 20 into the following forms:

$$\Delta x = (\mu_S - \frac{1}{2}\sigma_S^2)\Delta t + \sigma_S N(0,1)\sqrt{\Delta t} \tag{21}$$

$$\Delta y = (\mu_B - \frac{1}{2}\sigma_B^2)\Delta t + \sigma_B \hat{N}(1,0)\sqrt{\Delta t} \tag{22}$$

where

$\Delta x = \Delta \ln(S) = \ln(S/S_0)$

$\Delta y = \Delta \ln(B) = \ln(B/B_0)$

$N(0,1)$ and $\hat{N}(1,0)$ are correlated normal random variables with mean 0, variance 1 and correlation ρ.

Implementing Monte - Carlo simulation (MC) means executing equations 21 and 22 in a for loop in Excel VBA and updating the values of S and B at each time step until the end of the simulation horizon.

Simulating the payoff of each equity linked note is straightforward under the Black Scholes framework. Since under the Black and Scholes assumptions, volatility and interest rate are constant, equity linked note becomes a function of S only. Therefore, simulating the payoff of an equity linked note means computing the payoff of an equity linked note for a given simulated S at the end of the simulation horizon. Please refer to equations 1 and 6 for the payoff of PPN and BN.

3.2 Simulation Parameters

To implement the simulation, we need to make capital market assumptions for the expected returns, volatilities and the correlation of the simulated assets. In this thesis, I assume the markets are at long term equilibrium, and I use the estimates from Ibbotson Associates [9] for the total returns and volatilities of US equities and bonds:

Expected Total Return	
Stock	10.43%
Bond	5.44%
risk free	3.72%
Volatility	
Stock	20.31%
Bond	9.30%

Figure 15

For correlation, I compute the correlation of the monthly returns of the S&P500 Index and the BarCap US Agg Total Return Value Index for the past 20 years[26]:

Correlation Matrix

	Stock	Bond
Stock	1	0.2
Bond	0.2	1

Figure 16

Using these parameters, I simulate the asset values and calculate the payoffs of PPN and BN at the end of the investment horizon. I also assume domestic equities and domestic bonds investments involve 10bp and 20bp annual management fees respectively. For PPN and BN, 3.5% underwriting fees are incorporated when I calculated the theoretical terms of the products at inception in section 2.2.

[26]Source: Bloomberg

3.3 Risk Metrics

Traditionally, portfolio risk is measured by the standard deviation of the returns of the portfolio, and such risk measure is called volatility. However, the risk measure assumes the returns of the portfolio are normally distributed, which is not the case for equity linked structured products because the payoff profile of an equity linked structured products is asymmetric, i.e., there are elements of principal protection or enhanced upside potential which modifies the symmetric return profile of the underlying asset. As such, the returns of the equity linked structured products portfolio at the end of a five year horizon would not be normally distributed (i.e. not a bell curve), and thus the standard deviation of returns may not fully capture the risk of the ELN portfolio. In this analysis, I use Monte - Carlo techniques to simulate the return distribution and calculate the probability that the returns fall below certain threshold. As a result, I define "shortfall risk" as the expected loss, which is equal to the probability weighted return given that the portfolio's value falls below the risk free return of the initial investment. The risk free return is approximated by the T-bills return as shown in Figure 15. Mathematically, the shortfall risk is defined as:

$$ShortfallRisk = \int_{-\infty}^{r_f} F(x)dx \tag{23}$$

where:

r_f is the risk free rate

x is portfolio return

$F(.)$ is the cumulative distribution function of portfolio returns obtained from simulation

3.4 Portfolio Efficiency Metrics

In modern portfolio theory, portfolio efficiency is measured by the Sharp Ratio, which equals to the average excess return of the portfolio divided by the volatility of the portfolio. This ratio is a traditional metric for portfolio efficiency. However, since volatility cannot capture all the risks for portfolios with equity linked structured products, I decided to use shortfall risk (i.e. the expected loss) in determining the efficiency of the portfolio. Short fall is used for non normal distributions, structured products do not have normally distributed returns in comparison to equities. The shortfall risk measures losses at the tail of the distribution, where the the probability of loss is usually low. A portfolio is perceived to be efficient if it can maximize the expected gain and minimize the shortfall risk. Another way of saying this is that the most efficient portfolio is the one with maximum expected gain to shortfall risk ratio[27]. Therefore, in this paper, I define "portfolio efficiency" as: expected gain/shortfall risk. The higher the figure the more "efficient" the portfolio is. Mathematically, the expected gain and portfolio efficiency

[27] The expected gain/expected loss ratio is called omega. See References "An introduction to Omega, 2002" by Keating, Con and William F. Shadwick, See "Optimization of Conditional Value-at-Risk" by R.Tyrrell Rockafellar and Stanislav Uryasev, Sept 5, 1999.

are defined as:

$$ExpectedGain = \int_{r_f}^{\infty} F(x)dx \tag{24}$$

Portfolio Efficiency is calculated as the ratio of the Expected Gain over the Shortfall Risk.

3.5 Portfolio Performance Comparison

The table below compares the portfolio performance of the traditional, ELN I and ELN II portfolios from the simulations

	Traditional	ELN I: PPN	ELN II: BN
Expected Return	53.29%	53.37%	53.95%
Volatility	53.88%	54.90%	56.00%
Sharpe Ratio	0.62	0.61	0.61
Expected Excess Return	38.46%	38.54%	39.28%
Expected Shortfall	-5.20%	-5.20%	-5.37%
Portfolio Efficiency	7.39	7.41	7.32
Equities	60%	54%	54%
Bonds	40%	36%	36%
PPN		10%	
BN			10%

Figure 17

The result shows that the performances of the three portfolios are roughly the same. The expected total return of ELN II is a little higher than the other two portfolios, but it also has higher volatility. As a result, the Sharp Ratio of the three portfolios are essentially the same at 0.61. Another interesting point is that even though PPN and BN offer principal protection, they do not reduce the shortfall risk as expected because of the opportunity costs associated with these products. As a result, the portfolio efficiency of ELN I is about the same as the traditional portfolio, but the portfolio efficiency of ELN II is even lower than the traditional portfolio, which means that adding 10% of ELN into a portfolio does not improve the portfolio efficiency. I will explain the opportunity costs associated with these products in detail in the section below.

3.6 Optimal Portfolio Weights of Equity Linked Structured Products

In this section, I am going to increase the weight of equity linked structured product from 0% to 100% in each of the ELN portfolios to see if increasing the weight of equity linked structured products will improve the efficiency of a portfolio. Figure 18 shows the results of the analysis:

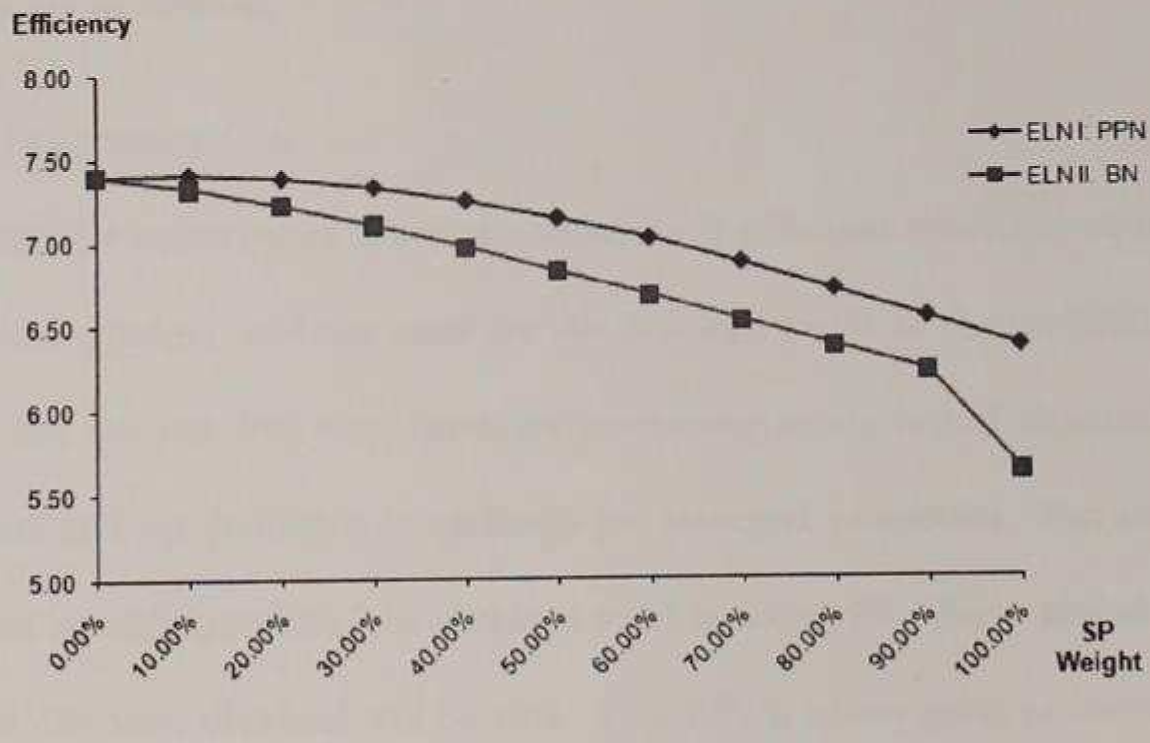

Figure 18

Figure 18 shows that increasing the weights of equity linked structured products in a portfolio will not necessarily improve the portfolio efficiency when the markets are at equilibrium. The result shows that the portfolio efficiency is at the maximum when the weight of PPN and BN is at 0%, which implies that adding PPN or BN does not improve the portfolio efficiency of the traditional portfolio. This graph is counter - intuitive to many investors because majority of investors believe that equity linked structured products with protection can reduce the risk and retain the upside potential of the portfolio, and thus enhance portfolio efficiency. However, the result shows that there is no benefit if the markets are at equilibrium, and I believe the reasons are as follows:

1. *Opportunity Cost*

There are opportunity costs associated with principal protected equity structured product, and the costs are the dividend yields of the underlying assets and the risk free rates.Investors purchasing equity linked structured products give up dividends in exchange for principal protection. For a diversified equities portfolio, the dividend yield is about 2%. For a period of 5 years, the total dividend will be 10%. This 10% is a very good protection cushion for the portfolio.

Instead of purchasing equity linked structured product to reduce the risk, alternatively investors can reduce their exposure in equities and move part of the investments into risk- free assets. When markets are at equilibrium the risk free rate is about 3.72%, which adds up to be 18.6% over a 5 year horizon. Therefore, even though ELN with protection can offer investors full principal or part of the principal back if the underlying assets fall in value, the opportunity costs are significant.

2. Diversification

Modern portfolio theory says that the risk of a portfolio can be reduced through diversification[28]. From Figure 16, returns of equities and bonds have very low correlation (0.2). During downturn, price returns of bonds tend to outperform equities, and the income from bonds and dividends from equities can compensate for the loss of equities. Reducing portfolio risk by investing in equity linked structured product seems to be inefficient because these structured products are correlated to both equities and bonds and would not provide further diversification. The result shows that though ELN with protection can reduce the risk of a portfolio, its effect is not as significant as diversification.

3. Markets tend to trend upward historically

Assuming markets are efficient and at equilibrium, history tells us that equity markets tend to grow and outperform bond markets over a long hori-

[28]See Appendix A6

zon as shown in Figure 15. That means there is no need to consider principal protection if investors have a long investment horizon because markets tend to grow over time on average. Any capital loss over the short term will be recovered if the portfolio is held over a long horizon.

3.7 Scenario Analysis

The analysis results in this section are based on the assumption that markets are at equilibrium. However, a lot of times markets are not at equilibrium and investors do face infrequent catastrophic risk over long investment horizon, which is not captured in the simulation analysis. To study the effect of infrequent catastrophic risk, we need to use historical scenario analysis.

In the following, I study how the ELN portfolio would have performed under the top 3 most severe financial crises - the current credit bubble, the dot com bubble and the oil crisis. The Great depression is excluded in this analysis because there is no bond data available back in the 1930's. In each of the cases, I calculated the total returns of the S&P500 Index and the BarCap US Agg Total Return Value Index for a five year period, which began 5 years before the crash of the S&P500 Index[29]. The results are as follows:

[29]For the oil crisis, the total returns of the 10y US Treasury is used as a proxy for the bond return because the BarCap US Agg Total Return Value Index is not available.

12/31/03 - 12/31/08 (Credit Crisis)

	Total Return	weight
S&P 500 Index	-10.49%	60.00%
Bond	25.52%	40.00%
Traditional portfolio	3.91%	
PPN	0%	
BN	0%	

SP Weight	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
ELN I: PPN	3.91%	3.52%	3.13%	2.74%	2.35%	1.96%	1.57%	1.17%	0.78%	0.39%	0.00%
ELN II: BN	3.91%	3.52%	3.13%	2.74%	2.35%	1.96%	1.57%	1.17%	0.78%	0.39%	0.00%

12/31/97 - 12/31/02 (Dot com Bubble)

	Total return	weight
S&P 500 Index	-2.88%	60.00%
Bond	43.87%	40.00%
Traditional portfolio	15.82%	
PPN	0%	
BN	0%	

SP Weight	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
ELN I: PPN	15.82%	14.24%	12.66%	11.07%	9.49%	7.91%	6.33%	4.75%	3.16%	1.58%	0.00%
ELN II: BN	15.82%	14.24%	12.66%	11.07%	9.49%	7.91%	6.33%	4.75%	3.16%	1.58%	0.00%

12/31/69 - 12/31/74 (Oil Crisis)

	Total return	weight
S&P 500 Index	-11.82%	60.00%
Bond	39.19%	40.00%
Traditional portfolio	8.58%	
PPN	0%	
BN	0%	

SP Weight	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
ELN I: PPN	8.58%	7.73%	6.87%	6.01%	5.15%	4.29%	3.43%	2.58%	1.72%	0.86%	0.00%
ELN II: BN	8.58%	7.73%	6.87%	6.01%	5.15%	4.29%	3.43%	2.58%	1.72%	0.86%	0.00%

Figure 19

In each case, the traditional portfolio (i.e. portfolio with structured products weight =0%) outperforms ELN portfolio with various structured products weights.

For example in the credit crisis (12/31/03 - 12/31/08), traditional portfolio return equals to 3.91%, but ELN I and ELN II returns range from 0%(100% ELN) to 3.53%(10% ELN). Similar results can be found during the Dot-com bubble (12/31/97 - 12/31/02) and the Oil Crisis (12/31/69 - 12/31/74). The results in this section are unexpected, and go against the perceived notion that structured products are great for investors especially in crisis scenarios. In other words, investors believe that because they are protected on the downside while maintaining their participation on the upside, there portfolio has increased. This perception is wrong - the reason is that bond has low correlation to equity and bond delivers positive returns through coupons to investors when the equity market experiences catastrophic risk during each of the crises, and such investment performances of the traditional portfolio are merits of diversification. On the other hand, even though ELNs provide principal protection, they under performed bonds because ELNs do not provide income to investors.There is no instance in which over the 5 year period the S&P500 was down more than 12%, which means that in all of the cases structured products with various weights in the portfolio returned 0. Increasing the weights of the structured products in the traditional portfolio does not increase efficiency - it does the opposite, it decreases efficiency since there are opportunity costs associated with integration structured products in a traditional portfolio. As a result, ELNs provide worse return performance than bonds during adverse scenarios.

4 CONCLUSION

While it is generally perceived that structured products with principal protection outperform other traditional investments strategies, after comparing the payoff distribution of portfolios with structured products and portfolios without structured products I conclude that this perception is wrong. In fact, integrating increasing weights of structured products in a traditional portfolio decreases the portfolio efficiency. My Monte Carlo simulations show that equity linked notes (ELN) with principal protection are not a good investment strategy if the benefits of structured products are evaluated from a portfolio perspective. I showed that equity linked structured product with protection feature is a good investment on a stand alone basis, but its benefits are overstated if it is viewed in a portfolio context. Specifically, in Section 2, I showed that investing in (ELN) results in a better risk/return reward than investing in underlying equity. However, investors need to be aware of the complicated market risk associated with these products in the secondary market, the constrained of holding the investment until maturity, the limited liquidity risk under a financial crisis and the issuer default risk.

In Section 3 I showed that ELN with protection is not able to enhance portfolio efficiency through both simulation and scenario analysis. The reason is that investors can reduce the risk of their portfolio through diversification, and ELN provides limited additional benefit to a well-diversified portfolio because investing in ELN has significant opportunity costs, and ELN is correlated to both equities and bonds and thus does not provide additional diversification. Therefore, ELN does not provide additional diversification to a well diversified traditional portfolio because of the significant opportunity costs and the fact that structured products are correlated to both stocks and bonds, thus there is no additional diversification resulting in no increase of its efficiency.

An interesting question for further research is then under what conditions ELN would increase portfolio efficiency and would be a better investment strategy over traditional assets. In my adverse scenario analysis, over the 5 year period, calculated returns on the S&P500 were not lower than 12%. Theoretically, one could create a situation in which the return on S&P500 is for example negative 25% over the 5 year period and analyze the return of the ELN portfolios with various weights. Another way of looking at this is, under what market conditions there will be a shift from investing in ELN portfolio being inefficient to being efficient.

One situation in which it could be wise to invest in ELN is when stocks and bonds become highly correlated, i.e. stock markets crash and many investment grade bonds experience default, but structured products issuers do not experience default. Under this scenario, structured products will outperform traditional portfolio. However, this scenario has not happened before and it's unlikely to happen in the future because if investment grade bonds default pick up, structured products issuers will be also likely to default.

5 Appendix

A1 Bonds

A bond is essentially an IOU issued by either a government, or government agency, or by a company. As such a bond is a promise to repay a fixed amount of money on a given date in the future and to make periodic interest payments (sometimes called coupon payments), which can be either fixed or variable.

Short dated bonds are similar to cash deposits in that they offer a return of capital in the near term. Longer maturity bonds have additional risk in that the repayment of capital becomes more dependant on the possibility of default by the bond issuer, which is generally higher the longer the maturity of the bond.

The price of a bond is dependant on the level of current interest rates and the coupon offered on the bond. If the coupon is variable, then the price will not be as sensitive to changes in the general level of interest rates.

The essential features of fixed rate is the such that as interest rates rise the price of the bond will fall, and vice versa. Meaning that when interest rates are up this results in bond prices to go down and when interest rates are down this results in bond prices to go up [23].

A2 Equities

Equities are the same as shares of a commercial company. Company shares simply represent the ownership of that company i.e. the company is owned by the shareholders.

Thus the terms equity market and stock market are interchangeable and simply mean the market where equities in different companies are bought and sold.

Equity holders, as owners of the company are generally entitled to a share of the profits. These may be paid to them in the form of dividends. A dividend is just the name for the payment received by a shareholder from the company representing a part of the profits made by the company. The exact dividend to be paid is determined by the board of directors.

Equities in large companies are often traded on the stock market. Such companies are called public companies. The share price is therefore a representation of the value of the company i.e. the market capitalization of a company is just the number of shares it has in issue multiplied by the share price [23].

A3 Options

There are two main kind of options - call option and put option.

A call option is a type of derivative that gives the holder the right, but not obligation, to purchase a set quantity of the underlying asset at a given price (the strike price) on or before a specified date (sometimes called the exercise date). Call options therefore benefit the owner if the price of the underlying rises.

If the underlying of the option is an index then typically the option will cash settle. This means that the seller will make a cash payment to the buyer equal to the difference between the final price of the index and the strike price multiplied by the notional size of the option.

So for example, the buyer of a call option on the S&P 500 index with a strike price of 4000 would receive a payment from the seller if the final index level was higher than 4000 on the maturity of the option. If the notional size of the option was \$10m then if the final index level was 4800 i.e. a 20% rise, then the seller would pay the buyer \$10m x 20% = \$2m. If the index was lower than 4000 at maturity then no payment would be made.

A put option is a type of derivative that gives the holder the right, but not obligation, to sell a set quantity of the underlying asset at a given price (the strike price) on or before a specified date (sometimes called the exercise date). Put options therefore benefit the owner if the price of the underlying falls.

If the underlying of the option is an index then typically the option will cash settle. This means that the seller will make a cash payment to the buyer equal to the difference between the final price of the index and the strike price multiplied by the notional size of the option.

So for example, the buyer of a put option on the S&P500 index with a strike price of 4000 would receive a payment from the seller if the final index level was lower than 4000 on the maturity of the option. If the notional size of the option was $10m then if the final index level was 3200 i.e. a 20% fall, then the seller would pay the buyer $10m x 20% = $2m. If the index was higher than 400 at maturity then no payment would be made.

In a nut shell - buying a call option allows the holder to profit from a rise in the share price and buying a put option on the other hand, provides a profit if the share price falls [13].

A4 Options Terminology

Options are referred to as "in the money", "at the money", or "out of the money". An "in the money" option would give the holder a positive cash flow if it were exercised immediately. Similarly, an "at the money" option would lead to zero cash flow if it were exercised immediately and an "out of the money" option would lead to negative cash flow if it were exercised immediately. If S is the stock price and K is the strike price, a call option is "in the money" when

$$S > K$$

"at the money" when

$$S = K$$

and "out of the money" when

$$S < K$$

A put option is "in the money" when

$$S < K$$

at the money when

$$S = K$$

and out of the money

$$S > K$$

An option will be exercised only when it is "in the money". In the absence of transaction costs, an "in the money" option will always be exercised on the expiration date if it has not been exercised previously.

The intrinsic value of an option is defined as the maximum of zero and the value the option would have if it were exercised immediately. For a call option, the intrinsic value is therefore:

$$max(S - K, 0)$$

For a put option the intrinsic value is

$$max(K - S, 0)$$

An "in the money" American option must be worth at least as much as its intrinsic value because the holder can realize a positive intrinsic value by exercising immediately. Often it is optimal for the holder of an "in the money" American option to wait rather than exercise it immediately. The option is then said to have "time value". The "time value" of an option is the part of the option's value that is derived out of the possibility of future favorable movements in the stock price.

Example: Suppose that the price of a call option with two months to maturity is \$3 when the stock price is \$30 and the strike price is \$28. Then the intrinsic value of the option is 30-28 =\$2 and the time value is 3-2=\$1. In general, the value of an option is zero when the option has reached maturity or it is optimal to exercise the option immediately.

The strike price of a call (put) option is the contractual price at which the underlier will be purchased (sold) in the event that the option is exercised. The last date on which an option can be exercised is called the expiration date. Options may allow for one of two forms of exercise:

With American exercise, the option can be exercised at any time up to the expiration date.

With European exercise, the option can be exercised only on the expiration date [13].

A5 Market Risks of Option

The effect on the price of a stock option of increasing one variable while keeping all others fixed* is summarized in the table below.

Variable	European call	European put	American call	American put
Current stock price	+	-	+	-
Strike Price	-	+	-	+
Time to maturity	?	?	+	+
Volatility	+	+	+	+
Risk-free rate	+	-	+	-
Dividends	-	+	-	+

* + indicates that an increase in the variable causes the option price to increase;
- indicates that an increase in the variable causes the option price to decrease;
? Indicates that the relationship is uncertain.

[13]

A6 Black and Scholes Formula Background

In 1973, Fisher Black and Myron Scholes published their groundbreaking paper "The pricing of options and corporate liabilities". Not only did this specify the first successful options pricing formula, but it also described a general framework for pricing other derivative instruments. This paper launched the field of financial engineering.

The Black-Scholes (1973) option pricing formula prices European put or call options on a stock that does not pay dividends or make other distributions. The formula assumes the underlying stock price follows a geometric

Brownian motion with constant volatility.

Values for a call price c or put p are:

$$c = s\Phi(d_1) - xe^{-rt}\Phi(d_2) \tag{25}$$

$$p = xe^{-rt}\Phi(-d_2) - s\Phi(-d_1) \tag{26}$$

where:

$$d_1 = \frac{\ln(s/x) + (r + \sigma^2/2)t}{\sigma\sqrt{t}} \tag{27}$$

$$d_2 = d_1 - \sigma\sqrt{t} \tag{28}$$

$s =$ the price of the underlying stock

$x =$ the strike price

$r =$ the continuously compounded risk free interest rate

$t =$ the time in years until the expiration of the option

$\sigma =$ the implied volatility for the underlying stock

$\Phi =$ the standard normal cumulative distribution function

Assumptions of the Black and Scholes Model:

1) The stock pays no dividends during the option's life

Most companies pay dividends to their share holders, so this might seem a serious limitation to the model considering the observation that higher dividend yields elicit lower call premiums. A common way of adjusting the model for this situation is to subtract the discounted value of a future dividend from the stock price.

2) European exercise terms are used

European exercise terms dictate that the option can only be exercised on the expiration date. American exercise term allow the option to be exercised at any time during the life of the option, making American options more valuable due to their greater flexibility. This limitation is not a major concern because very few calls are ever exercised before the last few days of their life. This is true because when you exercise a call early, you forfeit the remaining time value on the call and collect the intrinsic value. Towards the end of the life of a call, the remaining time value is very small, but the intrinsic value is the same.

3) Markets are efficient

This assumption suggests that people cannot consistently predict the direction of the market or an individual stock. The market operates continuously with share prices following a continuous Ito's process. To understand what a continuous Ito's process is, you must first know that a Markov process is "one where the observation in time period t depends only on the preceding observation." An Ito's process is simply a Markov process in continuous time. If you were to draw a continuous process you would do so without picking the pen up from the piece of paper.

4) No commissions are charged

Usually market participants do have to pay a commission to buy or sell options. Even floor traders pay some kind of fee, but it is usually very small. The fees that Individual investor's pay is more substantial and can often distort the output of the model.

5) Interest rates remain constant and known

The Black and Scholes model uses the risk-free rate to represent this constant and known rate. In reality there is no such thing as the risk-free rate, but the discount rate on U.S. Government Treasury Bills with 30 days left until maturity is usually used to represent it. During periods of rapidly changing interest rates, these 30 day rates are often subject to change, thereby violating one of the assumptions of the model.

6) Returns are lognormally distributed

This assumption suggests, returns on the underlying stock are normally distributed, which is reasonable for most assets that offer options [2].

A7 Modern portfolio theory

Modern portfolio theory was introduced by Harry Markowitz with his paper "Portfolio Selection," which appeared in the 1952 Journal of Finance. Thirty-eight years later, he shared a Nobel Prize with Merton Miller and William Sharpe for what has become a broad theory for portfolio selection.

Prior to Markowitz's work, investors focused on assessing the risks and rewards of individual securities in constructing their portfolios. Standard investment advice was to identify those securities that offered the best opportunities for gain with the least risk and then construct a portfolio from these. Following this advice, an investor might conclude that railroad stocks all offered good risk-reward characteristics and compile a portfolio entirely from these. Intuitively, this would be foolish. Markowitz formalized this intuition. Detailing a mathematics of diversification, he proposed that investors focus on selecting portfolios based on their overall risk-reward characteristics instead of merely compiling portfolios from securities that each individually have attractive risk-reward characteristics. In a nutshell, inventors should select portfolios not individual securities [3].

A8 Monte Carlo Simulation

For a long time, Monte Carlo was one of the best known locations for roulette and because a fair roulette wheel is one of the earliest random number generators, a branch of mathematics has been linked to Mediterranean seaside town.

Monte Carlo methods are a class of computational algorithms that rely on repeated random sampling to compute their results. Monte Carlo methods are often used when simulating physical and mathematical systems. Because of their reliance on repeated computation and random or pseudo-random numbers, Monte Carlo methods are most suited to calculation by a computer. Monte Carlo methods tend to be used when it is infeasible or impossible to compute an exact result with a deterministic algorithm.

Monte Carlo simulation methods are especially useful in studying systems with a large number of coupled degrees of freedom, such as fluids, disordered materials, strongly coupled solids, and cellular structures. More broadly, Monte Carlo methods are useful for modeling phenomena with significant uncertainty in inputs, such as the calculation of risk in business. These methods are also widely used in mathematics: a classic use is for the evaluation of definite integrals, particularly multidimensional integrals with complicated boundary conditions.

Monte Carlo simulation considers random sampling of probability distribution functions as model inputs to produce hundreds or thousands of possible outcomes instead of a few discrete scenarios. The results provide probabilities of different outcomes occurring [17],[18],[19].

A9 Derivatives

Derivatives are defined as financial instruments, the value of which depends on the value of their underlying instruments. Basic types of derivatives are futures contracts, forward contracts, options and swaps. A futures (or a forward) is a contract that obliges the holder to buy or sell an asset at a predetermined delivery price during a specified future time period. An option is a right to buy or sell an asset, while a swap is an agreement to exchange cash flows in the future according to a prearranged formula. A major breakthrough in the stock options pricing theory was made in 1973 by developing a valuation model known as the Black-Scholes model [20].

A10 Brownian Motion

In mathematics, the Wiener process is a continuous-time stochastic process named in honor of Norbert Wiener. It is often called Brownian motion, after Robert Brown. It is one of the best known Levy processes (stochastic processes with stationary independent increments) and occurs frequently in pure and applied mathematics, economics and physics. Brownian motion is the seemingly random movement of particles suspended in a liquid or gas (the physical phenomenon). The mathematical model used to describe such random movements is often called a particle theory. Brownian motion is among the simplest of the continuous-time stochastic (or random) processes, and it is a limit of both simpler and more complicated stochastic processes (random walk).

The Wiener process has applications throughout the mathematical sciences. In physics it is used to study Brownian motion, the diffusion of minute particles suspended in fluid. It also forms the basis for the rigorous path integral formulation of quantum mechanics and the study of eternal inflation in physical cosmology. It is also prominent in the mathematical theory of finance, in particular the Black-Scholes option pricing model. The mathematical model of Brownian motion has several real-world applications. An often quoted example is stock market fluctuations [9].

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